Cooperative Strategies and Achievable Rate for Tree Networks With Optimal Spatial Reuse

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***Abstract*—In this paper, a low-complexity cooperative protocol that significantly increases the average throughput of multihop upstream transmissions for wireless tree networks is developed and analyzed. A system in which transmissions are assigned to nodes in a collision free, spatial time division fashion is considered. The suggested protocol exploits the broadcast nature of wireless networks where the communication channel is shared between multiple adjacent nodes within interference range. For any upstream end-to-end flow in the tree, each intermediate node receives information from both one-hop and two-hop neighbors and transmits only sufficient information such that the next upstream one-hop neighbor will be able to decode the packet. This approach can be viewed as the generalization of the classical three node relay channel for end-to-end flows in which each intermediate node becomes successively source, relay and destination. The achievable rate for any regular tree network is derived and an optimal schedule that realizes this rate in most cases is proposed. Our protocol is shown to dramatically outperform the conventional scheme where intermediate nodes simply forward the packets hop by hop. At high signal-to-noise ratio (SNR), it yields approximately 66% throughput gain for practical scenarios.**

***Index Terms*—Fairness, relay channel, time division multiaccess, user cooperation, wireless networks.**

I. Introduction

THE demand for widespread Internet access over large urban areas draws an emerging interest both by the industry and the academic communities in designing high performance Wireless Mesh Networks (WMN) [1], [2]. WMN are expected to provide a low-cost but yet reliable and resilient high performance access network in which only few gateway nodes are connected to the wireline Internet. The rest of the nodes, the Transit Access Points (TAP), serve as relays which forward the client traffic (mobile and nonmobile) to/from the gateway. This technique allows hundreds of Internet users spanning large areas to share a single broadband connection.

A typical overlay routing topology of WMN is expected to look like a tree topology or forest of many trees, where the TAP's are the vertices and the gateways are the tree roots. All traffic that is destined for the wired network is transmitted by the clients to the nearby TAP. The aggregated traffic is relayed hop by hop until it reaches one of the gateways. All traffic from the wireline to any one of the clients follows the reverse path. Since each TAP is expected to serve many users, the TAPs are expected to be backlogged most of the time. In order to avoid contention between the clients and the TAP, it is assumed that the client-to-TAP transmissions and the TAP-to-TAP transmissions work on orthogonal channels. In this paper we concentrate only in the backbone formed by the TAP's which relay the traffic to the gateway (uplink transmission).

Recently several studies showed that existing Medium Access Control (MAC) protocols are not designed for multihop topologies. Deployment of existing protocols such as 802.11 in WMN can result in severe unfairness and low bandwidth utilization, i.e., when TAPs are backlogged, the backbone becomes a bottleneck and some of the TAPs get very poor throughput [3]–[6]. Since WMN promise to deliver high throughput to all users and in order for Internet Service Providers (ISP) to adopt the WMN as a solution for wide area broadband access network, an innovative solution is required to increase the average throughput.

In [7], the authors propose a scheduling that ensures per-client fairness and optimizes the bandwidth utilization in WMN. The solution assigns transmission rights to the links in a Spatial Time Division Multiple Access (STDMA) fashion and is collision free [8]. Whereas the proposed scheduling based on spatial reuse dramatically improves the average throughput in comparison with TDMA scheduling without spatial reuse, a natural question is whether higher throughput can be achieved by employing more sophisticated processing techniques.

Since WMN are relay networks, we consider multihop relaying technologies in order to improve network throughput. Besides the "classical three node relay channel" [9]–[12] and its extensions to multiple relays [13] or several sources [14]–[16], cooperative strategies have recently been considered for broader networks: 1) the "parking lot" topology which refers to a special case of a 1-ary tree (linear topology) with a flow originating from each node and terminating at the outermost node, [17]–[22]; 2) *ad hoc* networks with arbitrary or random topology [23]–[25]. Most of the aforementioned studies relate to networks where power optimization and interference cancelation are two of the main issues. Further, all these studies are

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limited to the gain achieved based on physical layer techniques. Recently some studies showed that cross layering architecture that combines the physical layer gain with proper scheduling and possibly rate control can further increase the throughput gain [2]; it was studied for mesh networks in downstream transmission in [26].

The aim of this paper is to develop and analyze low-complexity cooperative protocols that dramatically increase the average throughput of the upstream transmission for wireless mesh networks. Our analysis focuses on upstream communications in wireless mesh networks with a regular tree topology and can be extended to irregular tree topologies and downstream or bidirectional transmissions. We constrain the terminals to employ time-division transmission, i.e., the terminals cannot transmit and receive simultaneously. Furthermore, although previous works focused on the achievable rate region and the order of the throughput as a function of the number of nodes in the network, we derive the exact achievable rate with respect to the number of nodes for any regular tree network and propose deterministic schedules that realize this bound in most cases. The suggested solution assigns transmission rights to the nodes in a STDMA fashion and is collision free [7], [8], [27], [28]. In a conventional multi-hop routing each node forward its neighbors' traffic in addition to its own traffic [7], [27]–[29]. In the cooperative mode that we propose, each intermediate node receives information from *both* one-hop and two-hop neighbors and transmits only sufficient information such that the next upstream one-hop neighbor will be able to decode the packet. We name this local three-node cooperation "Turbo Relaying Protocol" (TRP).[1] Since a single node is transmitting within the interference range at the time, our strategy requires neither node synchronization at the sample level nor multiuser detection and can efficiently be implemented based on distributed low-density parity-check codes (LDPC) [16], [31], [32], or turbo-codes [33].

We compare TRP to the conventional noncooperative relay solution. In particular, we compare it to the method suggested in [7] which presents an optimal scheduling among the nodes in a spatial TD fashion which guarantees collision free transmissions. We also compare our scheme to an improved version of this solution where optimal power allocation is considered. We show that the optimal power allocation strategy slightly improves the scheme suggested in [7], however TRP dramatically outperforms both schemes; the throughput gains of TRP over the conventional scheme are above 66% at a signal-to-noise ratio (SNR) of 15 decibels for any connectivity degree.

The remainder of the paper is organized as follows: Section II describes the system model for tree network under consideration. In Section III, we derive the achievable rate for the conventional multihop routing, i.e., hop by hop relaying for a chain topology in a spatial time-division fashion [7] and a feasible schedule that realizes this rate is proposed. Section IV outlines our new node cooperation protocol. Achievable rate and optimal schedule that realizes this rate are determined. In Section V, these results are extended to tree networks with arbitrary connectivity degree. Finally, we draw conclusions in Section VI. In order to keep the flow of the paper all proofs are placed in the Appendices A–E.

[1]The term Turbo Relaying is not related to the channel error-correcting code family, the turbo-codes, invented by Berrou [30], but rather is based on the turbine engine principle. In turbocharged engines, the combustion air is already precompressed before being supplied to the engine. The engine aspirates the same volume of air, but due to the higher pressure, more air mass is supplied into the combustion chamber. Consequently, more fuel can be burnt, so that the engine's power output increases in relation to the same speed and swept volume.

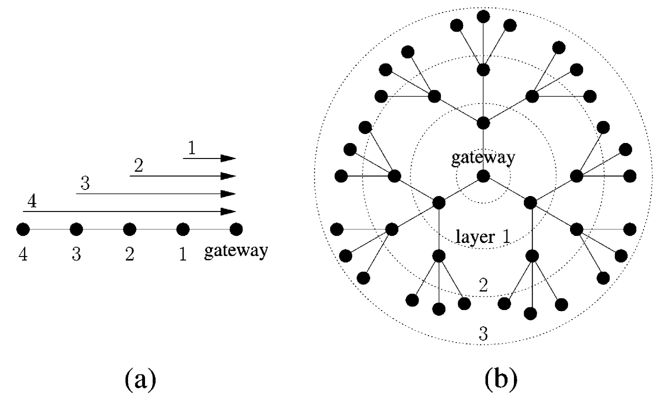

Fig. 1. Regular Tree Topologies. (a) 1-ary tree (connectivity degree of 2) also known as the parking lot topology. (b) Ternary tree with three levels (connectivity degree $m+1=4$). A link between two nodes means that these nodes are within transmission range of each other.

II. System Model

We model the wireless network as an m-ary tree topology where $m+1$ denotes the connectivity degree of any node. The root node of the tree represents the gateway and all the other nodes are TAP's that have no mobility. We define layer l as the set of nodes located l hops away from the gateway node. A link between two nodes means that these nodes are within transmission range of each other. Two examples of tree networks are depicted in Fig. 1: On the left, the parking lot topology which refers to the special case of a 1-ary tree (linear topology) with a flow from each node terminating at the right-most node. For this particular topology, the indices of the layers and the nodes coincide. On the right, a ternary tree with three layers where all nodes have a connectivity of $m+1=4$.

We utilize a baseband-equivalent, discrete-time channel model for the continuous-time channel. The distance d is normalized to the unit and we model the channel as

$$y_j[n] = x_i[n] + z_j[n] \tag{1}$$

where $x_i[n]$ is the transmitted signal by node i, and $y_j[n]$ is the destination received signal at the adjacent node j of node i. The variable $z_j[n]$ captures the effects of receiver noise and other forms of interference in the system. We model $z_j[n]$ as zero-mean mutually independent, circular symmetric, complex Gaussian random sequences with variance σ^2. The SNR of the transmission is defined as SNR$=P/\sigma^2$ with P transmission power.

With TRP, we also consider the received signal two hops apart from the transmitting node. Thus, the received signal two hops apart at node k can be expressed as

$$y_k[n] = x_i[n]/2^\gamma + z_k[n] \tag{2}$$

with γ channel pathloss exponent.

We focus on collision-free STDMA protocols that substantially simplify the signal processing algorithms at the receiver as in [7]. Moreover we assume that TAPs are placed in such a way that their transmission and interference ranges are the same for all TAPs and do not vary over time. In regular topologies such as in Fig. 1, it is equivalent to consider that the distances between any two adjacent connected nodes are equal, the channel attenuation coefficients are equal for all links and all TAPs are transmitting with equal power.

The signal-to-inference-plus-noise ratio (SINR) at any receiving node should be larger than a threshold τ depending on the performance of the receiver

$$\text{SINR} \geq \frac{P}{\sigma^2 + \sum_i \frac{P}{i^\gamma}} \geq \tau \tag{3}$$

where $i \in \{1, \ldots, N\}$ denotes the distance from the interfering node to the node of interest. For a wide variety of propagation channels, we have $2 \leq \gamma \leq 4$, [34], which gives an interference range of 3–5 times the transmission range. Assuming the use of omnidirectional antennas, we define the spatial reuse factor F as the minimum value of i in (3) plus one, which corresponds to the minimum number of hops between two nodes that can simultaneously transmit without interfering with each other, i.e., for hop by hop multihop relaying, a receiving node and an interfering node should be separated by at least $F-1$ hops. Under the assumption that all nodes are transmitting with fixed power P and have the same transmission and interference ranges unless mentioned otherwise, the spatial reuse factor F is a constant parameter for the network. With TRP, we also consider the received signal located two hops apart from a transmitting node. Thus, F should be increased by one from moderate to high SNR in order to keep the SINR below the threshold τ, i.e., $F^{\text{TRP}} = F + 1$. Whereas TRP can conceptually be extended to nodes located more than 2 hops apart from the transmitting node, the SINR is (very) small at these nodes for moderate values of $F, F \leq 10$. In this case, we expect very little throughput gain versus 2-hop-TRP for two reasons: 1) In order to keep a reasonably high SINR at nodes located more than 2 hops apart, we must significantly increase the spatial reuse factor, i.e., $F^{\text{TRP}} \gg F + 1$. This will cancel the throughput gain of TRP; 2) Current error-correcting codes such as LDPC codes or turbo-codes perform poorly at SNR lower than their theoretical threshold [30]. Therefore, attempting to decode the message at those nodes would deteriorate the performance instead of helping the transmission. Moreover, their convergence is very slow in such cases [31] and requires hundreds of iterations; this also reduces the throughput gain.

Finally, we constrain the terminals to employ time-division transmission, i.e., the terminals cannot transmit and receive simultaneously.

III. Achievable Rate for the Parking Lot Traffic Matrix ($m = 1$): The Noncooperative Case

We first consider a 1-ary tree ($m = 1$) also known as the parking lot topology shown in Fig. 1(a). The number of nodes N is equal to the depth of the tree L. In Section V, we extend our results to the general case $m, m > 1$. In Fig. 1(a), we also depicted the upstream flows for all nodes. The traffic load heavily depends on the position of the link in the network and has a significant impact on the cooperation strategy between the nodes, e.g., in Fig. 1(a) the final link carries four times the traffic of the left-most link.

A. No Cooperation Between the Nodes

In this section, we determine the achievable rate, i.e., the throughput per node normalized with respect to the occupied bandwidth, of an uplink transmission when "no cooperation" between the nodes is considered. Nodes are obliged to forward other nodes' messages on a fair basis share (e.g., round robin); however by no cooperation between nodes (in the sense of [14]), we assume that relaying is permitted only as a repeater's technique between neighbors. We assume that all nodes transmit with the same power P. The transmissions are according to a predetermined STDMA schedule which satisfies the spatial reuse condition (3), i.e., nodes which are less than F hops apart are not scheduled for transmission on the same time slot, however nodes which are more than F hops apart are allowed to be scheduled for transmission simultaneously. This is the common approach in link layer scheduling [7], [35].

Flow achievable rate R_k starting at node k is measured by the rate granted to the flow on its bottleneck link, i.e.

$$R_k = \sup_{t_i^k} \min_i t_i^k \mathrm{I}(X_i; Y_{i-1}) \tag{4}$$

where t_i^k is the transmission time granted to flow k on link $(i, i-1)$ and $\mathrm{I}(X_i; Y_{i-1})$ denotes the channel capacity on this link. All physical links having the same capacity in our model, we denote in the sequel $\mathrm{I}(X_i; Y_{i-1}), \forall i, i = 1, \ldots, N$ simply by C.

In our study we are interested in optimizing bandwidth allocation on a fair share basis, i.e., the total resources should be distributed such that the end-to-end rates are as equal as possible. Therefore the achievable rate is determined by the flow that gets the lowest rate

$$R = \sup_{s \in S} \min_{k \in \{1, \ldots, N\}} \{R_k^s\} \tag{5}$$

where the supremum is taken over all feasible schedules $s \in S$ where S is the set of all feasible schedules. R_k^s denotes the rate granted to flow k according to schedule s.

Theorem 1: For the upstream transmission in the parking lot network with all N nodes fully backlogged, with spatial reuse factor F, when all nodes transmit with the same power P and all links have the same capacity $C(P)$, the achievable rate $R(m = 1, F, N)$ is

$$R(1, F, N) = \frac{2C(P)}{F(2N - F + 1)} \tag{6}$$

where $C(P)$ equals for Gaussian sources [36]

$$C(P) = \frac{1}{2} \log_2(1 + P/\sigma^2) \tag{7}$$

with σ^2 noise variance.

The proof of the theorem is given in Appendix A. In the first part, we prove an upper bound on the achievable rate, i.e., we suggest a rate and prove that under no circumstances can we allocate a higher rate to *all* flows in the network. In the second part we prove that the suggested bound is tight, i.e., there exists

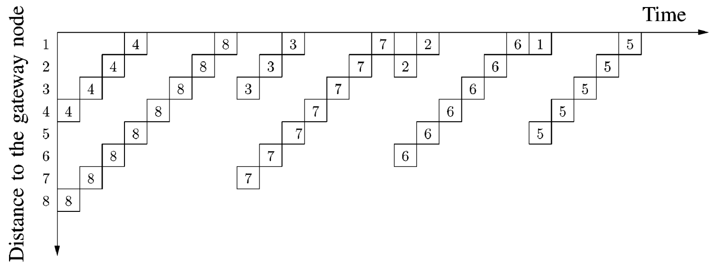

Fig. 2. Optimal schedule in a chain topology with spatial reuse factor $F = 4$ and $N = 8$ nodes. The cycle period to transmit one packet of all nodes to the gateway is equal to $F(2N - F + 1)/2 = 26$ time slots which corresponds to the achievable rate bound given by (6).

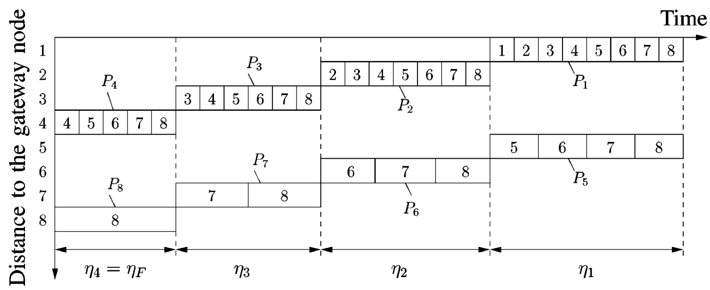

Fig. 3. Optimal schedule for the chain topology with a spatial reuse factor $F = 4$ and 8 nodes and with optimum power allocation. In comparison to Fig. 2, all previous idle time slots are now utilized. In order to avoid the congestion at the closer nodes, the farther the node, the less power it transmits.

a schedule that can realize the rate given by the bound, but there cannot be any schedule that can achieve a higher rate to *all* flows in the network. This result is an extension of [37, Lemma 3.1] for arbitrary spatial reuse factor in the case of fully backlogged nodes.

An example of a schedule which realizes the bound for eight nodes and spatial reuse factor $F = 4$ is shown in Fig. 2. On the horizontal and vertical axes, we show the slotted time and the node indices, respectively. A square in position (i, t) represents an active transmission from node i to node $i-1$ at lag t. In order to locate the path of the messages, the source node number is plotted in each square.

B. *Optimal Power Allocation (OPA)*

In the previous section, we assumed that all nodes transmit with fixed power P. According to the traffic pattern, the closer to the gateway the nodes, the more time they transmit, e.g., node 1 transmits $N/(N-1)$ times more than node 2 which transmits $(N-1)/(N-2)$ times more than node 3, etc. For node $i + jF, j \geq 1, i = 1, \ldots, F$, during a full cycle of the schedule, there are $(j-1)F + i$ idle time slots during which it could potentially transmit additional information without interfering with the other nodes. However, nodes $1, \ldots, F$ would not be able to forward this additional information to the gateway within the cycle period because there are no idle slots among the F closest nodes to the gateway in the optimal schedule. Nevertheless, higher throughput might be achieved if the farther nodes $i > F$ transmit also during the idle slots but with lower power than P. Assume that node i transmits with power P_i during t_i^k messages to its upstream node $i - 1$. The farther the nodes are to the gateway, the lower the transmission power and the longer the transmission duration, i.e., $P_N \leq P_{N-1} \leq \cdots \leq P_1$ and $t_N^k \geq t_{N-1}^k \geq \cdots \geq t_1^k$. In this case, the interference caused by node $i - F$ to node $i, i = F + 1, \ldots, N$ is higher than in the case of fixed transmit power. Thus, in order to maintain the same SINR (3) at node i as in the fixed power case, we should use a larger spatial reuse factor F. However, for sake of simplicity, we keep the same spatial reuse factor F as in the case with fixed power allocation and determine the power allocation $P_1^*, \ldots, P_N^*$ and the transmission durations $t_1^{k*}, \ldots, t_N^{k*}, \forall k$ that maximize the rate. Since (3) is usually not satisfied for F for this case, this rate is an upper bound of the achievable rate in a chain topology with optimal power allocation.

Theorem 2: For the upstream transmission in a chain topology of N fully backlogged nodes with spatial reuse factor F, the rate at any node, $R(1, F, N, \beta^*)$, is upper bounded by

$$R(1, F, N, \beta^*) \leq \frac{2(1+\beta^*)C(P)}{F(2N - F + 1)} \tag{8}$$

subject to

$$\sum_{k=1}^{N} \sum_{i=1}^{k} t_i^{k*} P_i^* \leq N(N+1)P/2. \tag{9}$$

where $\beta^* = t_1^{1*} \log(1 + P_1^*/\sigma^2)/\log(1 + P/\sigma^2) - 1$. Powers $P_1^*, \ldots, P_N^*$ are determined recursively as: $P_1^* = P_2^* = \cdots = P_F^*$ and $P_{i+F}^* = \gamma[\sigma_2 + \frac{P_i^*}{(F-1)^\gamma}], i = F + 1, \ldots, N$.

The proof of the theorem which includes the expressions of $t_1^{k*}, \ldots, t_N^{k*}, k = 1, \ldots, N$ is given in Appendix B.

Note that the achievable rate with fixed transmission power in (6) is a special case of (8) with $\beta = 0, P_1 = P_2 = \cdots = P_N = P$ and $t_1^k = \cdots = t_N^k, \forall k$, consequently $R(1, F, N, \beta^*) \geq R(1, F, N, 0)$, i.e., $\beta^* \geq 0$.

The same example as in Fig. 2 is depicted in Fig. 3 but this time with optimal power allocations instead of fixed transmission power. Compared to Fig. 2, all previous idle time slots are now utilized.

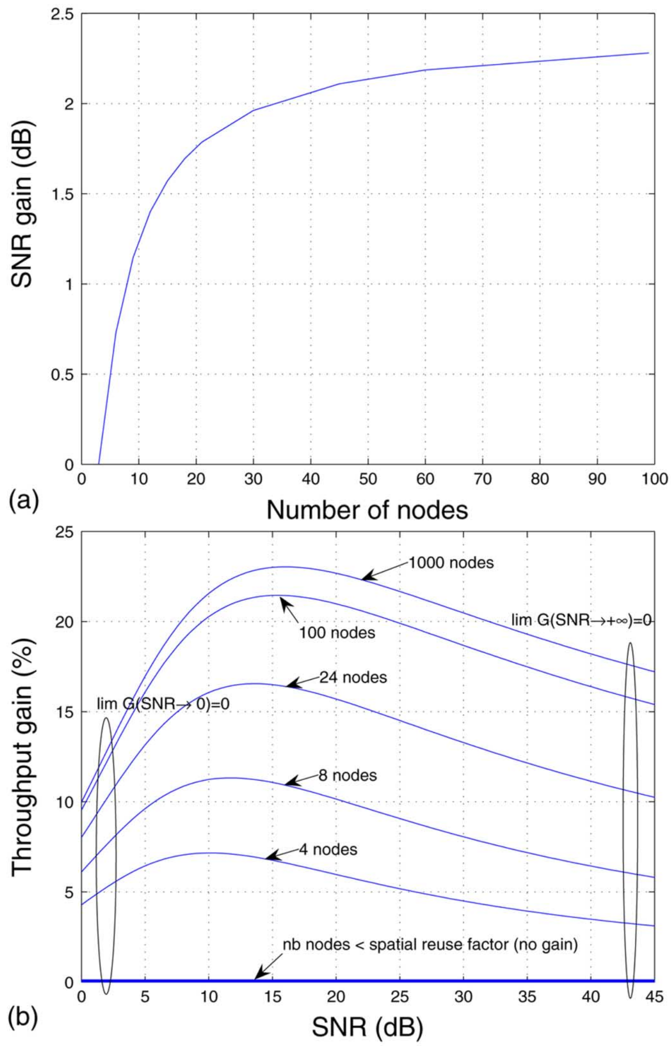

Fig. 4. SNR and throughput improvement with optimal power allocation (OPA) for the parking lot versus fixed power allocation scheme. The pathloss exponent equals 2 with spatial reuse factor $F = 4$. (a) SNR improvement given by (11) as a function of the number of nodes in the chain. The noise variance σ^2 is equal to 0.1. (b) Throughput improvement given by (10) as a function of SNR.

We illustrate the throughput and the SNR improvement with optimal power distribution versus fixed power allocation. We define the throughput gain $G_{\mathrm{OPA}}(1, F, N, \beta)$ as

$$G_{\mathrm{OPA}}(1, F, N, \beta^*) = \frac{R(1, F, N, \beta^*) - R(1, F, N)}{R(1, F, N)} = \beta^*. \tag{10}$$

Because (8) is usually not realizable, (10) is an upper bound of the theoretical gain. Similarly, the SNR gain $G'_{\mathrm{OPA}}(1, F, N, \beta^*)$ is upper bounded by

$$G'_{\mathrm{OPA}}(1, F, N, \beta^*) \leq \sigma^2 \cdot \frac{\left[(1 + P_1^*/\sigma^2)^{t_1^{1*}} - 1\right]}{P} - 1. \tag{11}$$

Fig. 4(a) and (b) shows the SNR gain $G'_{\mathrm{OPA}}(1, F, N, \beta^*)$ as a function of the number of nodes N in the chain and the upper bound of the throughput gain $G_{\mathrm{OPA}}(1, F, N, \beta^*)$ as a function of SNR for different chain sizes, respectively. The throughput gain with optimal power allocation becomes negligible at low and high SNR. For moderate SNR (10–15 dB), the gain is less than 25% even for a very large chain size (100, 1000 nodes). Accordingly, the gain in term of signal-to-noise ratio is small.

IV. Achievable Rate for the Parking Lot Traffic Matrix: The Cooperative Case

In the previous section, we determined the achievable rates for multihop uplink transmission in a chain topology with spatial reuse factor F. We assumed the common relaying strategy that consists of either transmitting its own information or repeating the messages received from the downstream node to the next node for both fixed and optimal transmission power.

In this section we propose a new cooperative relaying strategy which exploits the broadcast nature of wireless networks where the communication channel is shared among multiple adjacent nodes.

A. Introduction to Multi-Hop Transmission With Turbo-Relaying

There exist three main protocols for the classical three-node relay channel [10], [11], [38]: amplify-and-forward, decode-and-forward and compress-and-forward also referred to as quantize-and-forward. Although amplify-and-forward is the easiest to implement, a drawback of this approach is the noise amplification which occurs at the intermediate node [39]. Several compress-and-forward strategies based on Wyner-Ziv coding have been recently proposed in [39]–[41]. Particularly, those schemes exhibit promising gains when the relay node is close to the destination. In our channel model, the distance of all links are assumed to be the same, i.e., any intermediate node is located in the middle of its downstream node and the next node. Thus, we limit our study to decode-and-forward protocol which is the best strategy among the three protocols in that case. Recent implementations of decode-and-forward protocol based on LDPC codes perform as close as 0.6 dB to the theoretical limit [16], [31], [32] with single user computational complexity. The strategy in [31] is as follows: In the first time slot, a source n transmits part of the codeword which includes all information bits plus some redundant bits. Nodes $n-1$ and $n-2$ receive it but only node $n-1$ can decode it. Node $n-1$ re-encodes it with a high-rate LDPC code and transmits the redundant bits to node $n-2$. The rate of the LDPC code at the intermediate node is chosen such that node $n-2$ is able to decode the transmitted codeword based on the data received in both time slots. Perfect channel knowledge is assumed at the three nodes $n, n-1$, and $n-2$. Fig. 5 illustrates the strategy that we call Turbo Relaying. The arrows represent transmissions for three consecutive time slots.

Whereas this approach does not take full advantage of the beamforming gain as the source does not transmit together with the relay, it can be shown [42] that the loss versus the case where both source and relay transmit, does not exceed 10% for all configurations presented in this paper. Moreover, in order to fully benefit from the beamforming gain, both source and relay nodes have to be perfectly in phase and time synchronized at the sample level as in [14], [23], [31]. This synchronization task may introduce significant overhead. In our case, since only the relay

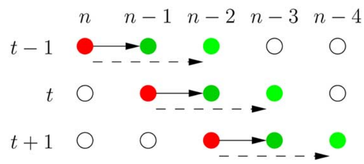

Fig. 5. Turbo Relaying strategy. Node n sends a packet such that node $n-1$ is able to decode it but node $n-2$ is not, since the distance between nodes n and $n-2$ is larger than the transmission range. However, node $n-2$ "listens" to this signal and stores it. Then, node $n-1$ transmits to node $n-2$ only the necessary information such that in addition to the previously stored received signal, node $n-2$ can decode the full packet. In the same manner, the process repeats itself until the root node is reached.

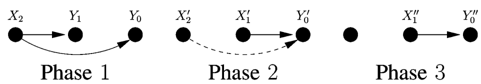

Fig. 6. Turbo Relaying strategy in a multihop transmission for a chain of three nodes (two nodes in addition to the gateway node). The dashed line in phase 2 refers to the classical "three node relay case" in TD fashion [43]. In this case, there is no phase 3 since the relay does not have to transmit its own data.

node transmits in the second phase, only frame synchronization is required, i.e., we assume that the relay detects the end of the transmitted message, decodes the message and starts to forward it to the destination within one or a few symbol periods. This delay becomes negligible as the message size tends to infinity.

Before detailing the calculation of the achievable rate based on this strategy for an arbitrary number of nodes, we treat the case of a chain with three nodes, i.e., nodes 1 and 2 and the gateway. This special case differs from the classical "three node relay channel" in the time-division (TD) mode, see for instance [31], [43], in 1) in the classical relay channel, the relay, i.e., node 1 with our notations, does not have its own information to transmit, and 2) Bounds for the achievable rate usually require perfect power control. In order to satisfy the spatial reuse constraint (3), we assume that all nodes transmit with fixed power P.

For a chain with three nodes in TD mode, the upstream transmission schedule is shown in Fig. 6.

In (5) of [31], it was shown that the achievable rate in TD mode for node 2 to transmit to node 0 is

$$R = \max_{\substack{t_1,t_2\\ t_1+t_2\le 1}} \min\{t_1 I(X_2;Y_1) + t_2 I(X_2';Y_0' \,|\, X_1') \quad t_1 I(X_2;Y_0) + t_2 I(X_2',X_1';Y_0')\}$$

where X_2 is the transmitted signal by node 2 during the first phase with duration t_1. Node 1 and the gateway node receive a noisy version of it, namely Y_1 and Y_0, respectively. X_2' and X_1' represent the signals transmitted simultaneously by nodes 2 and 1 during the second phase with duration t_2; Y_0' is the noisy superposition of those signals received at the gateway. In order to avoid node synchronization at the sample level, we assume that node 2 does not transmit during the second phase, i.e., $X_2' = 0$. Moreover, node 1 must also transmit its own packet during a third phase with duration t_3. Therefore, the achievable rate becomes

$$\begin{aligned} R = \max_{t_1,t_2,t_3} \min\{&t_1 I(X_2;Y_1)/(t_1+t_2+t_3)\\ &[t_1 I(X_2;Y_0) + t_2 I(X_1';Y_0')]/(t_1+t_2+t_3)\\ &t_3 I(X_1'';Y_0'')/(t_1+t_2+t_3)\}.\end{aligned}$$

The rate R is maximized when the three terms are equal [43], i.e.: $t_1 I(X_2;Y_1) = t_1 I(X_2;Y_0) + t_2 I(X_1';Y_0') = t_3 I(X_1'';Y_0'')$. Assuming fixed transmission power P for all nodes, we have $I(X_1'';Y_0'') = I(X_2;Y_1) = C(P)$ with $C(P)$ given by (7) which gives $t_3 = t_1$. According to our propagation model, $I(X_1';Y_0')$ is also equal to $C(P)$ and $I(X_2;Y_0) = C(P/2^\gamma)$. Therefore, t_2 can be expressed as: $t_2 = t_1[I(X_2;Y_1) - I(X_2;Y_0)]/I(X_1';Y_0') = t_1(1 - C(P/2^\gamma)/C(P))$.

Denote $\alpha = C(P/2^\gamma)/C(P)$. The achievable rate for nodes 1 and 2 becomes: $R = t_1 I(X_2;Y_1)/(t_1+t_2+t_3) = C(P)/(3-\alpha)$. The coefficient α is loosely bounded by: 0 (low SNR) $\le \alpha \le 1$ (high SNR) so the throughput R is bounded by:

$$C(P)/3 \le R \le C(P)/2.$$

For a chain of 2 nodes with single-hop relaying, the achievable rate is equal to $C(P)/3$. Therefore, the achievable rate based on TRP is always equal to or greater than the single-hop relaying case and the throughput improvement is up to 50%. This result is obtained without any power allocation optimization.

B. Turbo-Relaying Protocol (TRP) in a Chain Topology of n Nodes

In this section we extend the Turbo Relaying strategy to a chain of any size. Our main results are summarized in the next two theorems.

Theorem 3: Define the coefficient α as the ratio between the capacities of a single-hop transmission and a direct two-hop transmission

$$\alpha = \frac{C(P/2^\gamma)}{C(P)} = \frac{\log(1+P/2^\gamma\sigma^2)}{\log(1+P/\sigma^2)}. \tag{12}$$

The rate with Turbo-Relaying Protocol for parking lot topology is upper bounded by (13) shown at the bottom of the page with

$$\begin{aligned}\Phi_1(\alpha) &= ((F-1)[1-(-\alpha)^F])(1+\alpha) + (1+\alpha)(F+1)\\ &\quad - [1-(-\alpha)^{F+1}](-\alpha)^{N-2F+1}\end{aligned}$$

and

$$\Phi_2(\alpha) = \alpha \times ((N-F-1)(1+\alpha)(-\alpha)^{F-1} + [1-(-\alpha)^{F+1}]).$$

$$R(1,F,N,1,\alpha) \le \begin{cases} \frac{2(1+\alpha)^3 C(P)}{[(F+1)(2N-3F+2)+2F(F-1)](1+\alpha)^2+2\alpha\Phi_1(\alpha)}, & N > 2F+1\\ \frac{2(1+\alpha)^3 C(P)}{(2NF-F^2+F+2)(1+\alpha)^2+2N(1+\alpha)\alpha+2\alpha\Phi_2(\alpha)}, & N \le 2F+1 \end{cases} \tag{13}$$

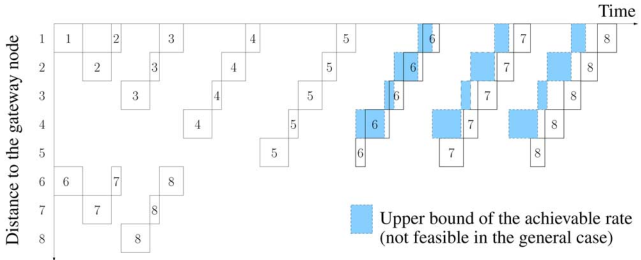

Fig. 7. Example of a schedule with Turbo Relaying Protocol for a chain topology with spatial reuse $F = 4$ and $N = 8$ nodes. This schedule realizes the achievable rate given by (14). Note that the transmission durations only depend on the relative distance between the transmitting nodes and the source node. For example, $t_6^6 = t_1^1, t_7^7 = t_2^2, t_6^7 = t_1^2$, etc.

Theorem 4: The rate $R(1, F, N, 1, \alpha)$ with Turbo-Relaying Protocol for parking lot topology is achievable if

$$R(1,F,N,1,\alpha) \leq \frac{2(1+\alpha)^3 C(P)}{[(F+1)(2N-F)](1+\alpha)^2 + 2\alpha\Phi_3(\alpha)} \tag{14}$$

with

$$\Phi_3 = (1+\alpha)(F+1) - [1-(-\alpha)^{F+1}](-\alpha)^{N-F}.$$

The proofs for both Theorems 3 and 4 are given in Appendix C.

In the general case there is no schedule that realizes the upper bound suggested in Theorem 3. Theorem 4 proposes a schedule that asymptotically with the number of nodes realizes the bound. In addition we prove that for small or moderate number of nodes, the achievable rate of the schedule is tight with the upper bound. Even though other schedules might achieve slightly better rate, the schedule that we suggest is simple in the sense that all nodes use the same strategy to forward their message regardless of their distance from the gateway.

An example of a schedule which realizes the bound for eight nodes with spatial reuse $F = 4$ is shown in Fig. 7.

The schedules in the non cooperative case and the cooperative case are similar with respect to the node transmission order but differ in the following two points: 1) the duration of each time slot differs with TRP depending on the relative distance between the transmitting node and the source node; 2) the spatial reuse factor needs to be increased by one unit with TRP in order to satisfy the spatial reuse constraint (3). Because all links have the same capacity and nodes do not interfere with each other, achieving the rate (14) over multiple links simultaneously with TRP as in the schedule on Fig. 7 is equivalent to independently achieve several times the rate of three-node relay channel. Information theoretic arguments about how to achieve this rate including codebook construction can be found in Appendix A in [43].

Define $G_{\text{TRP}}(1, F, N, 1, \alpha)$ the throughput gain with TRP versus the single-hop relaying case with fixed transmission power as

$$G_{\text{TRP}}(1,F,N,1,\alpha) = \frac{R(1,F,N,\alpha) - R(1,F,N)}{R(1,F,N)} \tag{15}$$

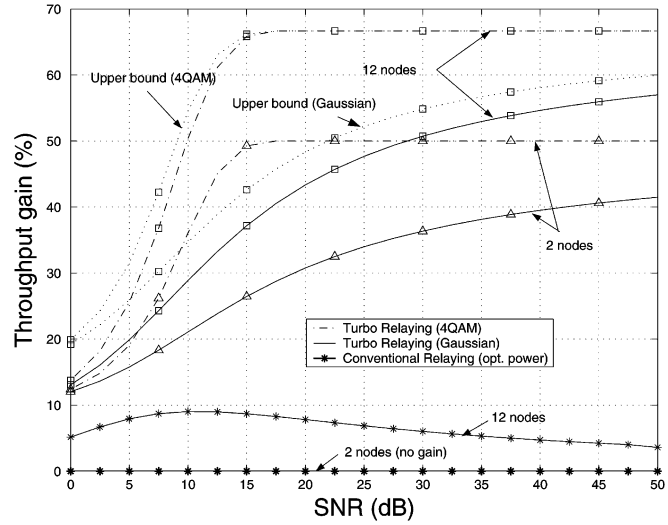

Fig. 8. Throughput gain for Turbo Relaying Protocol, given by (15). At high SNR for a chain of 12 nodes, TRP increases the throughput (14) by 66% compared to the noncooperative solution (6). Both strategies assume optimal spatial reuse. The channel pathloss exponent is equal to 2 for all cases with spatial reuse factor $F = 5$.

where $R(1, F, N)$ and $R(1, F, N, \alpha)$ are given by (6) and (14). The asymptotic gain improvements are summarized as follows

$$\lim_{N\to+\infty} G_{\text{TRP}}(1,F,N,1,\alpha) = (\alpha F - 1)/(F+1) \tag{16}$$

$$\lim_{\text{SNR}\to 0, N\to+\infty} G_{\text{TRP}}(1,F,N,1,\alpha) = \frac{F/2^\gamma - 1}{F+1} \tag{17}$$

$$\lim_{\text{SNR}, N\to+\infty} G_{\text{TRP}}(1,F,N,1,\alpha) = 1 - 2/(F+1). \tag{18}$$

In the low SNR regime, $\alpha \approx 1/2^\gamma$ with γ pathloss exponent. Assuming $\gamma = 2$, node cooperation gives some gain as long as the spatial reuse factor F is equal to or greater than 4 at any SNR for large networks. At high SNR and large spatial reuse factor F, TRP nearly doubles the throughput compared to the classical case. Indeed, $\alpha = 1$ corresponds to the fact that a node located two hops away from a transmitting node can directly decode the received packet without any additional information.

In (7), we considered Gaussian signals. However, it is interesting to evaluate the performance gains for constellations with finite alphabet. As the achievable rates are proportional to the link capacity $C(P)$, the throughput improvement given by (15) can be expressed for any modulation. In Fig. 8, we compare the throughput gain for Gaussian signal and quadrature amplitude modulated signal with four levels (4-QAM) between the three schemes: 1) conventional hop-by-hop relaying scheme with optimal scheduling (6), 2) hop-by-hop relaying scheme with optimal scheduling and optimal power allocation (8), and 3) TRP (14). Since α in (12) is larger for QAM-sources than for Gaussian signaling, the throughput gains are greater for 4-QAM signals than for Gaussian signals at any SNR, for any chain size and any channel pathloss exponent. Clearly, TRP with fixed transmission power outperforms both conventional hop-by-hop relaying schemes (6) with fixed power allocation or (8) with optimal power allocation at any SNR for any chain size. We also plotted the throughput gain based on the achievable rate upper bound given in (13). When the number of nodes in the chain is smaller than the spatial reuse factor, e.g., 2 nodes with spatial reuse factor $F = 5$ in Fig. 8, the upper bound (13) matches the achievable rate. For larger chain, e.g., 12 nodes in Fig. 8, the upper bound is tight and the gap between (14) and (13) does not exceed 6%.

V. The General Case $m > 1$: Regular Tree Network With Connectivity Degree of m

In this section, we extend the Turbo-Relaying Protocol to an arbitrary m-ary tree network with $m > 1$. The main differences with the parking lot topology are 1) spatial reuse can be exploited not only within a single flow's path but also through any other path of the tree, and 2) several nodes have to forward data toward to the same node in m-ary tree topology. As in the parking lot topology we first derive the achievable rate for the noncooperative case, and then proceed with the cooperative case (TRP).

A. No Cooperation Between Nodes

We first determine the achievable rate per node of an uplink transmission for an m-ary tree network when no cooperation between nodes is considered.

Theorem 5: For upstream transmission in a regular tree network with connectivity degree m, N fully backlogged nodes, spatial reuse F and fixed transmission power P, the rate at any node, $R(m, F, N)$, is achievable if (see (19) at the bottom of the page).

The proof is given in Appendix D. It has the same structure as the proof of Theorem 1, i.e., we first prove an upper bound to the achievable rate, and subsequently we present a schedule that realizes the bound.

B. Multihop Transmission With Turbo-Relaying

In this section, we extend the Turbo-Relaying Protocol to an m-ary tree network. We present the achievable rate based on the TRP strategy for any regular m-ary tree for upstream traffic, and suggest a schedule that realizes this rate. We focus on schedules that are simple to implement, in the sense that all nodes are using the same strategy. Although some schedules might achieve higher by exploiting idle slots, we expect modest gain as we showed for parking lot topology. Therefore, we omit the discussion about the achievable rate upper bound. We show that even if we use a simple schedule which is not optimal, the gain over the common hop-by-hop transmission is above 70% in most practical scenarios. The main result is summarized in the following theorem.

Theorem 6: For the upstream transmission in a regular m-ary tree network with depth L, all N nodes fully backlogged, and spatial reuse F, when all nodes transmit with the same power P and all single-hop links have the same capacity $C(P)$, the rate with TRP is achievable with (20) at the bottom of the page.

In order to prove that the rate presented in (20) is achievable with TRP, we suggest a schedule that attains this rate. We also show that using TRP without any modification (such modifications were proposed for the parking lot topology in Section C.A in order to determine an upper bound of the rate), the suggested schedule is optimal, i.e., we cannot increase the rate by scheduling the nodes or the flows differently. The proof is given in Appendix E.

C. Simulation Results

In this section, we illustrate the throughput improvement with TRP. For an m-ary tree with spatial reuse factor F and N nodes, the analytical throughput gain of TRP versus single-hop relaying is given by

$$G_{\mathrm{TRP}}(m, F, N, \alpha) = \frac{R(m, F, N, \alpha) - R(m, F, N)}{R(m, F, N)} \quad (21)$$

where $R(m, F, N)$ and $R(m, F, N, \alpha)$ are given in (19) and (20), respectively. Fig. 9 shows $G_{\mathrm{TRP}}(m, F, N, \alpha)$ as a function of the noise level for several connectivity degrees. In all cases, the tree depth is 8 with spatial reuse $F = 5$ and channel pathloss exponent $\gamma = 2$.

The throughput gain with TRP is greater than 50% at $\mathrm{SNR} = 12$ dB for 4-QAM signaling. We also observe at high SNR the same asymptotic gain for Gaussian sources at high SNRs.

$$R(m, F, N) = \begin{cases} 2C(P)m(m-1)^2/\{[N(m-1)+m][m(F-2)+2] \\ \times(m-1) + 2m^{F/2}(1-2m) + 2m^2\}, & \text{if } F \text{ is even} \\ (m-1)^2 C(P)/\{[N(m-1)+m](m-1)(\lceil F/2\rceil - 1) \\ -m^{\lceil F/2\rceil} + m\}, & \text{if } F \text{ is odd.} \end{cases} \quad (19)$$

$$R(m, F, N, \alpha) \leq \begin{cases} m(m-1)^2(1+\alpha)^2(1+m\alpha)C(P)/\{[N(m-1) \\ +m][(m\lfloor F/2\rfloor + 1)(1+\alpha)(1+\alpha m) \\ \times(m-1) + (-\alpha)^{L+1-\lfloor F/2\rfloor}(m-1)^2[1+\alpha-m\alpha \\ \times(1-(-\alpha)^{\lfloor F/2\rfloor})]] - (1+\alpha)^2 \\ \times[m^{\lfloor F/2\rfloor+1}(2m-1) - m^2]\}, & \text{if } F \text{ is odd} \\ (m-1)^2(1+\alpha)^2(1+m\alpha)C(P)/\{[N(m-1) \\ +m][(F/2)(1+\alpha)(1+\alpha m)(m-1) \\ +(-\alpha)^{L+2-F/2}[1-(-\alpha)^{F/2}](m-1)^2] \\ -(1+\alpha)^2(m^{F/2}-1)\}, & \text{if } F \text{ is even.} \end{cases} \quad (20)$$

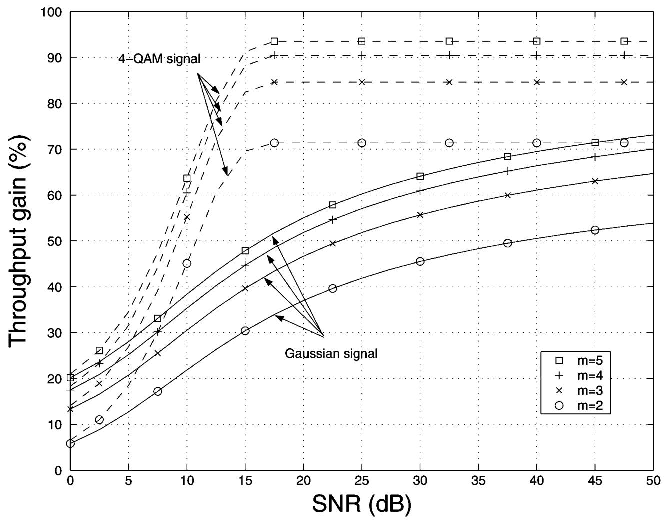

Fig. 9. Throughput improvement in (21) for Turbo Relaying Protocol (20) versus conventional hop by hop relaying (19) as a function of the signal-to-noise ratio for several connectivity degrees m. Both strategies assume optimal spatial reuse. The throughput gain with TRP is greater than 50% at SNR $= 12$ dB. We observe the same asymptotic gain with Gaussian sources for high SNR. Throughput improvement is higher as the connectivity degree of the tree increases for 4-QAM and Gaussian sources. For $m = 5$, the gain approaches 100% for 4-QAM signaling for SNR greater than 15 dB. In all cases, the channel pathloss exponent is equal to 2 with spatial reuse $F = 5$.

Throughput improvement is higher as the connectivity degree of the tree increases for 4-QAM and Gaussian sources. This is due to better spatial reuse within the layers and the subtrees if m is large as discussed in Theorem 6. For $m = 5$, the gain approaches 100% for 4-QAM signaling for SNR greater than 15 dB.

VI. Conclusion

We proposed the Turbo-Relaying Protocol to increase each node's throughput of upstream transmission in tree topologies. We determined the achievable rate for any regular tree networks based in this protocol. Whereas power optimization leads to a near-zero throughput gain compared to the basic case of multihop hop-by-hop transmission with fixed power allocation, we showed that our approach can achieve over 66% throughput gain for any regular tree with any connectivity degree. For large connectivity degree and sufficient SNR, the gain approaches 100%. Whereas the cooperative strategy that we propose can be extended to different transmission ranges and/or the interference ranges, theoretical analysis depends heavily on the choice of topology and the propagation model. In the case of time-varying channel, it might be interesting to follow the approach in [11] for single relay or in [44] for multiple relays where the outage probability behavior is analyzed in Rayleigh fading environment.

Appendix A
Proof of Theorem 1

First we show an upper bound on the achievable rate, i.e., we suggest a rate and prove that under no circumstances can we allocate a higher rate to *all* flows in the network. In the second step we prove that the suggested bound is tight and can be realized, i.e., a lower upper bound cannot be found.

A. Upper Bound of the Achievable Rate

A flow achievable rate is based on the minimum rate granted to the flow over all links. By taking the minimum over only a partial set of the links which includes only the first F hops (closest to the gateway), flow k achievable rate is upper bounded by

$$R_k \leq \begin{cases} \min\{t_F^k \mathrm{I}(X_F; Y_{F-1}), t_{F-1}^k \mathrm{I}(X_{F-1}; Y_{F-2}), \ldots, \\ t_1^k \mathrm{I}(X_1; Y_0)\}, \qquad F \leq k \leq N \\ \min\{t_k^k \mathrm{I}(X_k; Y_{k-1}), t_{k-1}^k \mathrm{I}(X_{k-1}; Y_{k-2}), \ldots, \\ t_1^k \mathrm{I}(X_1; Y_0)\}, \qquad 1 \leq k < F. \end{cases} \tag{22}$$

Since the achievable rate is determined by the flow that gets the lowest rate, and since $\mathrm{I}(X_i; Y_{i-1}) = C(P) \forall i \in \{1, 2, \ldots, F\}$, the achievable rate upper bound is determined according to $\min_{k, i \leq F}\{t_i^k\}$. Denote τ as the total time granted to all flows on the first F nodes (nodes $1, 2, \ldots, F$), i.e., $\tau = \sum_{k=1}^{N} \sum_{i=1}^{\min\{k,F\}} t_i^k$. Recall that the spatial reuse factor is F, i.e., two nodes that are separated by less than F hops cannot transmit simultaneously. Accordingly, based on (22) the rate upper bound is obtained by splitting τ equally between all t_i^k, i.e.

$$\begin{aligned} t_i^k &= \frac{\tau}{\sum_k \sum_{i=1}^{\min\{k,F\}} 1} \\ &= \frac{2\tau}{F(2N - F + 1)}, \quad \begin{array}{l} k = 1, \ldots, N \\ i = 1, \ldots, \min(k, F). \end{array} \end{aligned}$$

Since all t_i^k and $\mathrm{I}(X_i; Y_{i-1})$ are equal, R_k according to (22) is the same for all k. Note that this maximization guarantees a total fairness among the flows. We normalize the total schedule duration to one time unit. Since τ is the transmission duration of a partial set of nodes for which no spatial reuse can be exploited, it cannot exceed the complete schedule duration, i.e., $\tau \leq 1$. Accordingly, the achievable rate is upper bounded by

$$R \leq \frac{2C(P)}{F(2N - F + 1)}. \tag{23}$$

This completes the proof of the first part. It is important to note that without suggesting a schedule that can realize this bound, it is not clear that the bound is tight. Next we present a schedule that realizes the bound.

B. Lower Bound on the Achievable Rate

Since the achievable rate is determined based on the schedule that yields the highest rate in (5), a rate corresponding to any specific schedule is a lower bound to the achievable rate. We describe the schedule and determine its rate.

Due to spatial reuse all nodes which are F hops apart can transmit simultaneously without interference but nodes which are less than F hops apart cannot transmit simultaneously. Hence, it takes F time slots to forward each packet by exactly one hop, i.e., forward one packet from node N to node $N-1$, one packet from node $N-1$ to node $N-2, \ldots,$ one packet from node 2 to node 1 and one packet from node 1 to the gateway. Thus it takes $N - F$ time slots to forward one packet from nodes $N, N-1, \ldots, N-F+1$ to nodes $F, F-1, \ldots, 1$ respectively, and one packet from the rest of the nodes to the gateway. Since there is no spatial reuse, the time needed to forward the remaining packets from nodes $F, F-1, \ldots, 1$ to the gateway is $F, F-1, \ldots, 1$ time slots, respectively. Hence the total time required to bring one packet from each node to the gateway is $(N-F)\cdot F + \frac{(F+1)\cdot F}{F} = \frac{F(2N-F+1)}{2}$. This process can repeat itself cyclically.

Assuming that all messages carry $C(P)$ information bits per channel use, and since each flow transmits at least one message to the gateway, the achievable rate realized by the schedule per flow is greater than $\frac{C(P)}{(F(2N-F+1)/2)}$, i.e., $R_k \geq \frac{2C(P)}{(F(2N-F+1))}$. Accordingly the achievable rate is bounded from below (5), i.e.

$$R \geq \frac{2C(P)}{(F(2N-F+1))}.$$

Since the lower bound matches the upper bound in (22), the suggested schedule attained the achievable rate.

Remark: In [45] it was shown that in a network in which the attenuation between any two nodes is strictly positive, i.e., every node is within interference range of any other node in the network, max-min fairness yields equal rates to all flows. However, note that the difference in our model is that nodes which are F hops apart can simultaneously transmit without interfering with each other, i.e., neither of two transmitters which are $F+1$ hops apart and scheduled for transmission on the same time slot, will have any rate gain by the decrease of the other transmitter's rate.

Appendix B
Proof of Theorem 2

We first determine the achievable rate per node for arbitrary power allocation and arbitrary coefficient β defined as

$$\beta = \min_{\substack{i,k\\ 1\leq k\leq N\\ i\leq k}} \frac{t_i^k C(P_i) F(2N-F+1)}{2C(P)} - 1 \tag{24}$$

where t_i^k and P_i satisfy (9).

Proposition 1: For the upstream transmission in a chain topology of N fully backlogged nodes with spatial reuse factor F and given power allocation $\{P_1, P_2, \ldots, P_N\}$ subject to constraint (9), the rate at any node, $R(1,F,N,\beta)$, is achievable if

$$R(1,F,N,\beta) \leq \frac{2(1+\beta)C(P)}{F(2N-F+1)}. \tag{25}$$

Proof: According to (4), the flow rate is defined as

$$R_k = \min_{0\leq i\leq k} t_i^k C(P_i). \tag{26}$$

For fixed power scheme, we found in Theorem 1 that the flow rate R_k is achievable if $R_k \leq 2C(P)/F(2N-F+1)$. In (24), the coefficient β is defined as the ratio between the achievable rate for flow k given by (26) and the achievable rate without power allocation optimization (6). By taking the minimum flow rate over all values of $k, k=1,\ldots,N$, we obtain (25).

We wish now to determine the optimal transmission powers $P_1^*, \ldots, P_N^*$ and the optimal transmission durations $t_1^{k*}, \ldots, t_k^{k*}, k=1,\ldots,N$ that maximize the achievable rate (26). Our main result is summarized in the following proposition.

Proposition 2 (Optimal Power Allocation): Define $\eta_i, i=1,\ldots,F$ as

$$\eta_i = \max\left\{\sum_{k=i}^{N} t_i^k, \sum_{k=i+F}^{N} t_{i+F}^k, \ldots, \sum_{k=i+\lfloor (N-i)/F\rfloor (F-1)}^{N} t_{i+\lfloor (N-i)/F\rfloor F}^k\right\} \tag{27}$$

with normalization: $\sum_{i=1}^{F}\eta_i = 1$. Consider an upstream transmission of the unitary tree network with power and time-sharing allocated among the relays in such a way that the throughput per node is optimal subject to (9). Then, the power allocation/time-sharing optimization problem is equivalent to the following optimization problem:

$$\{P_1^*, \ldots, P_N^*, t_1^{k*}, \ldots, t_N^{k*}, k=1,\ldots,N\} = \arg\min_{\eta_1,\eta_2,\ldots,\eta_F,P_1} \frac{\eta_1}{2}\log(1+P_1/\sigma^2) \tag{28}$$

subject to (9) and the following constraint:

$$\sum_{i=1}^{F} \eta_i W_1^{\frac{\eta_1(N-i+1)}{\eta_i N}} \cdot \frac{1-W_1^{-\frac{\eta_1}{\eta_i}}}{1-W_1^{-\frac{F\eta_1}{N\eta_i}}} \leq N(2N-F+1)/2 + N(N+1)P/2\sigma^2 \tag{29}$$

where we define: $W_1 = 1 + P_1/\sigma^2$.

Powers $P_2^*, \ldots, P_N^*$ are derived from P_1^* and $\eta_1^*, \ldots, \eta_F^*$ as follows:

$$P_i^* = \sigma^2(1+P_1^*/\sigma^2)^{\frac{\eta_1^*(N-i+1)}{\eta_i^* N}} - 1, \qquad i=1,\ldots,F \tag{30}$$

and

$$P_{jF+i}^* = \sigma^2(1+P_i^*/\sigma^2)^{\frac{N-(jF+i)+1}{N-i+1}} - 1, \quad j=1,\ldots,\lfloor (N-i)/F\rfloor - 1, i=1,\ldots,F. \tag{31}$$

Proof: By keeping the same spatial reuse constraint as in the fixed transmission power case, all nodes $i, i+F, \ldots, i+F\lfloor (N-i)/F\rfloor, i=1,\ldots,F$ can transmit simultaneously but not with nodes $i', i'+F, \ldots, i'+F\lfloor (N-i')/F\rfloor, i'=1,\ldots,F, i'\neq i$. Node $i+jF$ has to transmit the information of the $N-i-jF$ nodes located below itself in addition to its own information and is given at most the duration η_i to do so. Based on Jensen's inequality [10], the rate at node $i+jF$ is maximized if the power is spread as much as possible, i.e., if node $i+jF$ is transmitting during η_i. Therefore, we have: $(N-i-jF+1)t_{i+jF}^k = \eta_i, i=1,\ldots,F, j=0,\ldots,\lfloor (N-i)/F\rfloor, k=i,\ldots,N$. The total power constraint becomes

$$\eta_1(P_1 + P_{F+1} + \cdots + P_{F\lfloor (N-1)/F\rfloor + 1}) + \cdots + \eta_F(P_F + P_{2F} + \cdots + P_{F\lfloor N/F\rfloor}) \leq N(N+1)P/2. \tag{32}$$

The relationship between t_{i+jF}^k and $t_{i'+jF}^k$ can be expressed as: $t_{i+jF}^k(N-i-jF+1)/\eta_i = t_{i'+j'F}^k(N-i'-j'F+1)/\eta_{i'}$. Achievable flow rate R_k of node k is maximized if all terms in (4) are equal, i.e., if $t_i^k C(P_i) = t_j^k C(P_j), i=1,\ldots,N, j=$

$1,\ldots,N$ with OPA. By combining the two latest expressions, we have

$$W_i = W_1^{\frac{\eta_1(N-i+1)}{\eta_i N}}, \quad i=1,\ldots,F$$
$$W_{i+jF} = W_i^{\frac{N-i-jF+1}{N-i+1}}, \quad j=0,\ldots,\lfloor (N-i)/F \rfloor$$

that are equivalent to (30) and (31), respectively with $W_i = 1 + P_i/\sigma^2, \forall i$. By combining them with (32) we obtain (29). Note that we reduce the number of parameters from $N+N(N+1)/2 = \mathcal{O}(N^2)$ in the initial optimization problem to $F+1$ parameters, P_1 and $\eta_1,\ldots,\eta_F$. Therefore, the computational complexity of the power optimization does not depend on the number of nodes. To the best of our knowledge, there is no analytical solution to (28) subject to (29) for general case. A numerical solution can be found with Matlab for example.

The upper bound given by (25) of the achievable rate becomes

$$R(1,F,L,\beta^*) \leq \frac{2(1+\beta^*)C(P)}{F(2N-F+1)} \tag{33}$$

where $\beta^* = \eta_1^* \log(1+P_1^*/\sigma^2)/N\log(1+1/\sigma^2) - 1$.

Appendix C
Proofs of Theorems 3 and 4

We first determine an upper bound of the achievable rate as for the non cooperative case (Theorem 3). Contrary to the non cooperative case, there is generally no schedule that realizes this bound over TRP. Here, we propose a simplified schedule for practical considerations that asymptotically with the number of nodes realizes the bound. We also show that the achievable rate of the schedule is tight with the upper bound for small or moderate number of nodes.

A. Upper Bound of the Achievable Rate

Since TRP is based on decode-and-forward scheme, each intermediate node has to decode the message w_k of node k. Accordingly, node $k-1$ can decode the message reliably as long as the length of the coded sequence $X_k(w_k)$ is large and $R_k^k < t_k^k I(X_k;Y_{k-1})$. Based on the same arguments suggested for the three-node relay case e.g., Appendix A in [43]), node $j-1, j<k$ can decode the message reliably as long as all previous nodes are able to decode the message and in addition it receives enough information from its previous one and two hop neighbors, hence

$$R_j^k < \min\left\{R_k^k, R_{k-1}^k, \ldots, R_{j+1}^k, t_{j+1}^k I(X_{j+1};Y_{j-1}) + t_j^k I(X_j;Y_{j-1})\right\}.$$

Therefore, according to (4), the flow k achievable rate R_k is

$$R_k = \min\left\{t_k^k I(X_k;Y_{k-1}), \min_{1\leq j\leq k-1}\left\{R_j^k, t_{j+1}^k I(X_{j+1};Y_{j-1}) + t_j^k I(X_j;Y_{j-1})\right\}\right\}. \tag{34}$$

In order to derive an achievable rate upper bound for TRP, we determine a lower bound on the cycle duration of any feasible schedule, where a cycle is defined as the minimum time needed to transmit at least one unit of information from each node to the gateway. We lower bound the cycle duration by considering the minimum transmission time needed only over the first $F+1$ hops instead of F hops in the non cooperative case in order to verify the interference model in (3). Part of the information originally sent by nodes that are located farther than $F+1$ hops away from the gateway, reaches node F directly with TRP when node $F+2$ transmits. Theoretically, node F can receive all information directly from node $F+2$ without node $F+1$ transmitting even one bit. Hence, in order to compute the minimum duration over the first $F+1$ hops, we have to take into account the information already received by node F directly from node $F+2$ when $F+2$ was transmitting to node $F+1$.

In order to determine the amount of information that node $F+2$ already transmitted to node F, we define the two sets of flows ϕ_1 and ϕ_2 as $\phi_1 = \{1,2,\ldots,(N-F-1)\}$ and $\phi_2 = \{(F+2),(F+3),\ldots,N\}$, respectively. Additionally, denote τ_1 and τ_2 as the minimum time needed to transmit the information of the set of nodes ϕ_1 all the way to the gateway and the minimum time needed to transmit the information of the set of nodes ϕ_2 to node $F+1$ respectively. For symmetry reason, $\tau_1 = \tau_2$, i.e., the minimum time required to transmit one unit of information from each flow in ϕ_1 all the way to the gateway is the same as the minimum time required to transmit one unit of information from each flow in ϕ_2 all the way to node $F+1$. Furthermore, node $F+2$ transmits the same amount of information as node 1 during τ_1. Assuming optimal schedule, there are no extra idle time slots available for node $F+2$ to transmit extra information to node F during τ_1. However τ_1 is only the amount of time needed to forward the flows in set ϕ_1 to the gateway and the flows in set ϕ_2 to node $F+1$. The information from flows $\{N-F+1, N-F+2,\ldots,N\}$ that do not belong to Φ_1 still needs to be forwarded from node $F+1$ to the gateway. As mentioned above, some of the information already reached node F over TRP after τ_1. However, since we are considering an optimal schedule, no information that belongs to these flows has reached beyond node F, i.e., during τ_1 nodes $\{1,2,\ldots,F+1\}$ did not transmit any information belonging to flows $\{N-F+1, N-F+2,\ldots,N\}$. Denote τ_3 as the minimum amount of time needed to forward these latter flows to the gateway. Assuming that during τ_1, node $F+1$ receives all information which belongs to flows in set ϕ_2, node $F+2$ can further exploit during τ_3 any additional time it can transmit without interfering with node 1 while the latter node is transmitting. Indeed, node $F+2$ can transmit to node F additional information that belong to flows $\{N-F+1, N-F+2,\ldots,N\}$ that node F has not received during τ_1. Therefore, the duration of τ_3 can be (slightly) shortened. Nevertheless, the time that node $F+2$ can transmit the additional information without interfering with the other nodes is limited to the time that node 1 is transmitting to the gateway. Hence, the first time that node $F+2$ can transmit this information to node F is when node 1 is transmitting flow $N-F$ to the gateway, the next time is when node one is transmitting flow $N-F+1$, etc. Moreover, the capacity of the link between node 1 and the gateway is significantly higher than the capacity between node $F+2$ and F. Therefore, for practical scenarios, node $F+2$ does not have sufficient time to transmit all information to node F. Nevertheless, in order to provide a lower bound on the duration of τ_3, we shall assume that node F receives all the information of flows $N-F+1, N-F+2,\ldots,N$ directly from node $F+2$, i.e., during τ_3 node $F+1$ does not need to transmit any information to node F. As previously explained for realistic scenarios, no schedule can realize this bound in the general case.

In (35) (shown at the bottom of the page) we summarize the upper bound of the transmission rates. We consider only the first $F+1$ layers where no spatial reuse can be used. Therefore, these terms are the dominant terms in (34) that determine the cycle period

For flow $k, 1 \leq k < F$, we count all transmissions since there is no spatial reuse. For flow $k, F \leq k < N-F$, we consider only transmissions over the last $F+1$ hops as they belong to set ϕ_1, and for flow $k, N-F+1 \leq k < N$, which belongs to set ϕ_2 we consider only the transmissions over the last F hops.

In order to determine an upper bound on the achievable rate, i.e., a lower bound on a full cycle period, we determine the durations t_j^k that minimize (35), $1 \leq j \leq F+1, 1 \leq k \leq N$. We start by computing the minimum duration t_k^k needed for node $k, k = 1, \ldots, N$ to send its own information, i.e., the minimum duration to sustain a rate R_k^k of at least $C(P)$ per time slot normalized to the unit. Thus, each term in the minimization in (34) should be greater than or equal to $C(P)$. Then, we have: $t_k^k C(P) \geq C(P)$, i.e., $t_k^k \geq 1$. The second term $t_k^k I(X_k; Y_{k-2}) + t_{k-1}^k I(X_{k-1}; Y_{k-2}) = t_k^k C(P/2^\gamma) + t_{k-1}^k C(P)$ should also be greater than $C(P)$ to guarantee a flow rate of $C(P)$. Hence, $t_{k-1}^k \geq 1-\alpha$, where α denotes the ratio between the achievable rate of one hop and two hop transmissions, i.e., $\alpha = \frac{C(P/2^\gamma)}{C(P)}$. Continuing in the same manner with the other terms in (34), we get $t_j^k \geq \sum_{i=0}^{k-j} (-1)^i \alpha^i$.

In order to bound τ_1 from below, we sum the time needed to transmit the information from flows in set ϕ_1 through the last $F+1$ hops. Based on the previous calculations of $t_{j,j-1}^k, \tau_1$ is lower bounded by the second equation at the bottom of the page.

Note that we distinguished in the first sum between the cases $N < 2F+1$ and $N \geq 2F+1$, since in the first case the number of flows that can benefit from the "extra" idle slots is smaller than $F-1$, and depends on N. In the latter case, the number of flows that "push" extra information over TRP to node F is constant and is equal to $F-1$. Also note that for completeness we should have also differentiated the case where $N \leq F+1$, where no flows can facilitate any extra idle slots, i.e., the upper index in the first sum should have been: $\min\{\max\{N-F+1, F+1\}, N\}$. However, since in the latter TRP and the optimal case coincide, we omit this term for simplicity and the achievable is given by (4).

Next we determine a lower bound on τ_3. Assuming that all messages $N-F, N-F+1, \ldots, N$ already reached node F during τ_1, the minimum amount of time needed to forward flows $N-F+1, N-F+2, \ldots, N$ from node F to the gateway, is

$$\tau_3 \geq \sum_{k=\max\{N-F+2, F+2\}}^{N} \sum_{j=1}^{F} \sum_{i=0}^{F-j} (-\alpha)^i = \begin{cases} \frac{(F-1)F}{1+\alpha} + \frac{\alpha(F-1)(1-(-\alpha)^F)}{(1+\alpha)^2}, & N > 2F+1 \\ \frac{(N-F-1)F}{1+\alpha} + \frac{\alpha(N-F-1)(1-(-\alpha)^F)}{(1+\alpha)^2}, & N \leq 2F+1. \end{cases}$$

For the same reason as above, we omitted the case where $N \leq F+1$ which coincides with TRP. Combining both lower bounds on τ_1 and τ_3, the total time τ needed to achieve a rate of $C(P)$ per hop between any node and the gateway is lower bounded by (36) also shown at the bottom of the page. Hence, the achievable rate is upper bounded by

$$R \leq \frac{C(P)}{\tau} \tag{37}$$

where τ is given by (36).

$$R_k \leq \begin{cases} \min\big\{t_k^k I(X_k; Y_{k-1}), t_k^k I(X_k; Y_{k-2}) \\ \quad + t_{k-1}^k I(X_{k-1}; Y_{k-2}), \ldots \\ \quad t_2^k I(X_2; Y_0) + t_1^k I(X_1; Y_0)\big\} \quad 1 \leq k < F, \\ \min\big\{t_{F+1}^k I(X_{F+1}; Y_{F-1}) + t_F^k I(X_F; Y_{F-1}) \\ \quad t_F^k I(X_F; Y_{F-2}) + t_{F-1}^k I(X_{F-1}; Y_{F-2}), \ldots, \\ \quad t_2^k I(X_2; Y_0) + t_1^k I(X_1; Y_0)\big\}, \quad F \leq k \leq N-F \\ \min\big\{t_F^k I(X_F; Y_{F-1}) \\ \quad t_F^k I(X_F; Y_{F-2}) + t_{F-1}^k I(X_{F-1}; Y_{F-2}), \ldots, \\ \quad t_2^k I(X_2; Y_0) + t_1^k I(X_1; Y_0)\big\}, \quad N-F+1 \leq k \leq N. \end{cases} \tag{35}$$

$$\tau_1 \geq \sum_{k=1}^{\max\{N-F+1, F+1\}} \sum_{j=1}^{\min\{F+1, k\}} \sum_{i=0}^{k-j} (-\alpha)^i = \begin{cases} \frac{(F+1)(2N-3F+2)}{2(1+\alpha)} + \frac{\alpha(1+\alpha)(F+1)+\left(1-(-\alpha)^{F+1}(-\alpha)^{N-2F+2}\right)}{(1+\alpha)^3}, & N > 2F+1 \\ \frac{(F+1)(F+2)}{2(1+\alpha)} + \frac{\alpha\left[(1+\alpha)(F+1)+\alpha\left(1-(-\alpha)^{F+1}\right)\right]}{(1+\alpha)^3}, & N \leq 2F+1. \end{cases}$$

$$\tau = \tau_1 + \tau_3 \leq \begin{cases} \frac{(F+1)(2N-3F+2)+2F(F-1)}{2(1+\alpha)} + \frac{\alpha(F-1)(1-(-\alpha)^F)}{(1+\alpha)^2} \\ \quad + \frac{\alpha(1+\alpha)(F+1)+(1-(-\alpha)^{F+1})(-\alpha)^{N-2F+2}}{(1+\alpha)^3}, & N > 2F+1 \\ \frac{2NF-F^2+F+2}{2(1+\alpha)} + \frac{(N-F-1)(-\alpha)^{F+1}}{(1+\alpha)^2} \\ \quad + \frac{\alpha N(1+\alpha)+\alpha^2(1-(-\alpha)^{F+1})}{(1+\alpha)^3}, & N \leq 2F+1. \end{cases} \tag{36}$$

B. Achievable Rate

We consider a schedule for which all flows are forwarding the messages in a similar fashion, and all nodes are using the same strategy regardless of their distance from the gateway. Contrary to the approach that we considered to determine the upper bound, we do not exploit the idle slots in layer $F+2$ during τ_3 to transmit extra information to node F. We show that even so, the rate of the suggested schedule is very close to the achievable rate upper bound. As pointed out in the previous subsection, no schedule can realize the upper bound (14) even if the idle slots are exploited.

The schedule is based on the time sharing of the full cycle duration among the first $F+1$ nodes (nodes $1, 2, \ldots, F+1$) such that any of these nodes has sufficient time to forward to the gateway one message from all nodes located below in the chain. We show that during this cycle, by exploiting spatial reuse, it is possible to schedule the forwarding of one message from each of the nodes $F+2, \ldots, N$ to reach node $F+1$.

With spatial reuse factor $F+1$, nodes

$$k + i(F+1), \quad 1 \le k \le F+1, 0 \le i \le \left\lfloor \frac{N}{F+1} \right\rfloor$$

can transmit simultaneously without interfering with each other hence they are scheduled to transmit simultaneously. The transmission duration of the forwarded information with TRP depends on the distance (in term of hops) between the source node and the transmitting node. In order to maximize the spatial reuse, nodes that are scheduled to transmit simultaneously are coordinated with each other such that they transmit simultaneously information which belongs to nodes that are the same hop distance from them, i.e., nodes $k+i(F+1), 1 \le k \le F+1,\ 0 \le i \le \lfloor \frac{N}{F+1} \rfloor$ transmit simultaneously the information belonging to nodes $k+i(F+1)+h, 0 \le h \le N-i(F+1)-k$: Nodes $i(F+1)+1, i=0, \ldots, \lfloor (N-1)/(F+1) \rfloor$ are scheduled to transmit their own messages simultaneously; they also transmit simultaneously the message of their one-hop neighbor, etc. The time granted to each transmission is determined according to the transmission durations $t_i^k, i, k \le F+1$ that have been computed in the previous subsection, i.e., node $k, k \le F+1$ is granted $\sum_{i=k}^{N} t_i^k$ time units. Accordingly, the cycle duration is $\sum_{k=1}^{F+1} \sum_{i=k}^{N} t_i^k$ time units. The transmission duration t_i^{i+k} needed by node $i+k$ to forward the information originally sent by node $i+k$ is $\sum_{l=0}^{(i+k)-i} (-\alpha)^l = \sum_{l=0}^{k} (-\alpha)^l$ and depends only on the difference between i and $i+k$. In this schedule, all nodes transmit and are allocated sufficient time to forward the information from all nodes located below in the chain. Moreover, any two nodes that transmit simultaneously are at least $F+1$ hops apart, which means that they are out of interference range.

Appendix D
Proof of Theorem 5

The proof of Theorem 1 follows the same guideline as the proof of Theorem 1 for chain topology. In the first part we give an upper bound to the achievable rate, and in the second part we present a schedule that realizes the bound.

A. Upper Bound of the Achievable Rate

We adopt the following notations: The gateway node is denoted as node 0. The remaining nodes are numbered using two indices, the first referring to the depth of the node in the tree, i.e., its distance from the gateway, and the second to the position of the node within the layer. The numbering within layers always starts from node 1 and continues counterclockwise within the layer. Node 1 in the first layer is chosen arbitrarily; in the other layers, node 1 is defined as the left-most descendant of node 1 from the previous layer. The set of nodes $n_{(i,:)}$ denotes all nodes in the ith layer, i.e., all nodes i hops away from the gateway node. We denote t_i^k the time per cycle granted to node i for transmitting the information belonging to flow k. Occasionally we use a single index to specify a node (we keep the index which refers to the layers omitting the index within a layer). Since we focus on the upstream traffic with a tree overlay routing topology, it is not necessary to also refer the index of the upper layer. Indeed, the flow of any node k passes through a unique node per layer before to reach the gateway. Accordingly, X_F^k denotes the message of node k which is forwarded by node F during time t_F^k. Y_{F-1}^k denotes the corresponding received signal received by the neighbor node in layer $F-1$.

As in the parking lot topology, in order to determine an upper bound on the achievable rate we examine the achievable rate over a subset of links noticing that the achievable rate for the whole network can only be equal or smaller. We distinguish between the cases with F even and F odd.

In the case of odd F, we define the set of nodes S_{odd} as the set of all nodes belonging to the first $(\lceil \frac{F}{2} \rceil - 1)$ layers, i.e., $S_{\text{odd}} = \{n_{(i,:)} \,|\, 1 \le i \le \lceil \frac{F}{2} \rceil - 1\}$. The number of hops between any two nodes in the set is less than F, i.e., there is no spatial reuse within nodes in set S_{odd} as illustrated in Fig. 10 for $F=7$.

Accordingly, the achievable rate per node is upper bounded by (38a) at the bottom of the page.

Denote K_{odd} as the total number of time slots in (38a), i.e., the sum of all $\{t_i^k \,|\, 1 \le k \le N, 1 \le i \le \min[k, \lceil F/2 \rceil - 1]\}$.

$$K_{\text{odd}} = N + \sum_{l=2}^{\lceil F/2 \rceil - 1} \left(N - \sum_{j=1}^{l-1} m^j \right) = \frac{(\lceil F/2 \rceil - 1)((m-1)N + m)(m-1) + m - m^{\lceil F/2 \rceil}}{(m-1)^2}.$$

$$R_k \le \begin{cases} \min \left\{ t^k_{\lceil \frac{F}{2} \rceil - 1} \mathrm{I}\left(X^k_{\lceil \frac{F}{2} \rceil - 1}; Y^k_{\lceil \frac{F}{2} \rceil - 2} \right), \right. \\ \quad \left. t^k_{\lceil \frac{F}{2} \rceil - 2} \mathrm{I}\left(X^k_{\lceil \frac{F}{2} \rceil - 2}; Y^k_{\lceil \frac{F}{2} \rceil - 3} \right), \ldots, t_1^k \mathrm{I}\left(X_1^k; Y_0^k \right) \right\}, & k \notin S_{\text{odd}} \\ \min \left\{ t_k^k \mathrm{I}\left(X_k^k; Y_{k-1}^k \right), t_{k-1}^k \mathrm{I}\left(X_{k-1}^k; Y_{k-2}^k \right), \ldots, \right. \\ \quad \left. t_1^k \mathrm{I}\left(X_1^k; Y_0^k \right) \right\}, & k \in S_{\text{odd}}. \end{cases} \tag{38a}$$

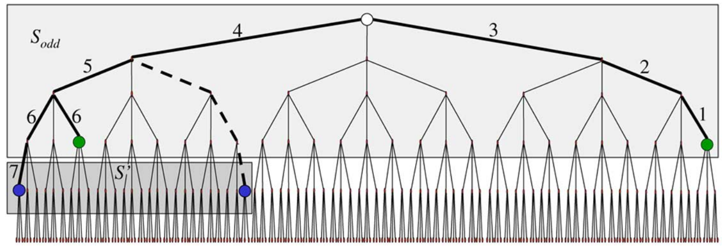

Fig. 10. Illustration of the sets $S_{\rm odd}$ and S' for a ternary tree. The set $S_{\rm even}$ is equal to $S_{\rm odd}\cup S'$. Two cases: a) The spatial reuse factor F is odd ($=7$). The number of hops between any two nodes in the set $S_{\rm odd}$ is 6 at most which is strictly less than F, i.e., there is no transmission spatial reuse within nodes in the set $S_{\rm odd}$. b) The spatial reuse factor F is even ($=8$). The number of hops between any two nodes in the set $S_{\rm even}$ is 7 at most which is also strictly less than F, i.e., there is no transmission spatial reuse within nodes in the set $S_{\rm even}$ either.

Denote $t_{\rm odd}$ as the minimum time slot duration per flow received by any node in $S_{\rm odd}$

$$t_{\rm odd}=\min_{1\le k\le N, i\in S_{\rm odd}} t_i^k \le \frac{\tau}{K_{\rm odd}},$$

where τ denotes the total duration of a schedule cycle. The cycle duration is greater than or equal to the aggregate time granted to the nodes in set $K_{\rm odd}$ (no spatial reuse within $S_{\rm odd}$). Since the rate is determined by the flow with the minimum rate, we bound the achievable rate by

$$R\le t_{\rm odd}\cdot C\le \frac{\tau}{K_{\rm odd}}\cdot C.$$

In the case of even F, define the set S' as the group of nodes belonging to layer $\frac{F}{2}$ and which are descendants of node $n_{1,1}$, i.e., $S'=\{n_{\frac{F}{2},j}\,|\,1\le j\le m^{\frac{F}{2}}\}$, and denote $S_{\rm even}=S'\cup S_{\rm odd}$. The distance between any two nodes in S' is smaller than or equal to twice the number of hops to node $n_{1,1}$, i.e., $2(\frac{F}{2}-1)$, and the distance between a node in S' to the farthest node in $S_{\rm odd}$ is the distance from node $i\in S'$ to node 0 plus the distance from node 0 to a node in layer $\frac{F}{2}-1$, i.e., $\frac{F}{2}+\frac{F}{2}-1$. Therefore there is no spatial reuse in $S_{\rm even}$. An example with $F=8$ is shown in Fig. 10.

Denote by $B_{(j,k)}$ the subtree rooted at node $n_{(j,k)}$, i.e., node $n_{(j,k)}$ itself and all of its descendants in the tree; e.g., B_0 is the complete tree and $B_{(1,1)}$ is the subtree with root node $n_{(1,1)}$. Denote by $B_{(j,k)}(l)$ the set of nodes belonging to subtree $B_{(j,k)}$ that are l hops from node $n_{(j,k)}$. $B_{(j,k)}(l)$ defines the intersection between the subtree $B_{(j,k)}$ and the set of all nodes in layer $j+l$ on the original tree, i.e., $B_{(j,k)}(l)\equiv B_{(j,k)}\cap n_{(j+l,:)}$. Finally denote by $|B_{j,k}|$ the total number of messages transmitted by all nodes in set $B_{j,k}$. Therefore, we get (38b) shown at the bottom of the page, where $\delta_{k\in B_{(1,1)}}$ equals one if node k is a descendant of node $(1,1)$ and zero otherwise.

The additional number of time slots to add to $K_{\rm odd}$ in order to get $K_{\rm even}$ is the number of messages that pass through the nodes in S', which is $\sum_{j=\frac{F}{2}}^{L} m^{j-1}=\frac{m^L-m^{\frac{F}{2}-1}}{m-1}$. Using the relation $m^L=\frac{(m-1)N+m}{m}$ with N the total number of nodes in the tree with depth L, the number of time slots that should be added to $K_{\rm odd}$ is hence $\frac{(m-1)N+m-m^{\frac{F}{2}}}{m(m-1)}$ and

$$\begin{aligned}K_{\rm even}&=K_{\rm odd}+\frac{(m-1)N+m-m^{F/2}}{m(m-1)}\\&=\{(m-1)((m-1)N+m)(m(F/2-1)+1)\\&\quad+m^{F/2}(1-2m)+m^2\}\cdot\frac{1}{m(m-1)^2}.\end{aligned}$$

Using the same reasoning as with odd F and by replacing odd with even in the notations, $t_{\rm even}$ and the achievable rate R become

$$t_{\rm even}=\min_{1\le k\le N, i\in S_{\rm even}} t_i^k\le\frac{\tau}{K_{\rm even}}$$

and

$$R\le t_{\rm even}\cdot C\le\frac{\tau}{K_{\rm even}}\cdot C.$$

Normalizing the total schedule duration to one time unit $\tau=1$, the achievable rate is upper bounded by

$$R\le C(P)\cdot t_{\rm even/odd}\le\frac{C(P)}{K_{\rm even/odd}}.$$

This completes the first part of the proof in which we determined an achievable rate upper bound. Next, we present a schedule that can realize the bound. Note that by suggesting a schedule that achieves the rate $C(P)/K$, we only bound the rate from below (achievable rate lower bound) since we have not shown that there cannot be a schedule that attains a higher rate. However, since the upper bound matches the lower bound, the rate is exactly $R=C(P)/K$.

$$R_k\le\begin{cases}\min\Big\{\delta_{k\in B_{(1,1)}}t^k_{\lceil\frac{F}{2}\rceil}\mathrm{I}\left(X^k_{\lceil\frac{F}{2}\rceil};Y^k_{\lceil\frac{F}{2}\rceil-1}\right),\\ \quad t^k_{\lceil\frac{F}{2}\rceil-1}\mathrm{I}\left(X^k_{\lceil\frac{F}{2}\rceil-1};Y^k_{\lceil\frac{F}{2}\rceil-2}\right),\ldots,t^k_1\mathrm{I}\left(X^k_1;Y^k_0\right)\Big\}, & k\notin S_{\rm even}\\ \min\left\{t^k_k\mathrm{I}\left(X^k_k;Y^k_{k-1}\right),t^k_{k-1}\mathrm{I}\left(X^k_{k-1};Y^k_{k-2}\right),\ldots,\right.\\ \quad\left.t^k_1\mathrm{I}\left(X^k_1;Y^k_0\right)\right\}, & k\in S_{\rm even}\end{cases}\tag{38b}$$

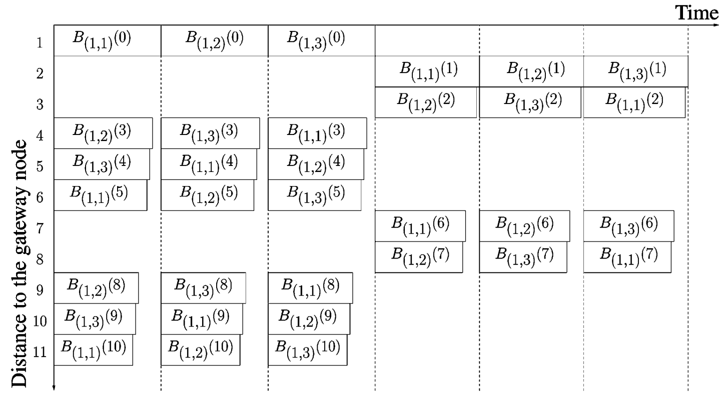

Fig. 11. An example of optimal schedule for ternary tree with odd spatial reuse factor as determined in (39). The depth of the tree is $L = 11$ and the spatial reuse $F = 5$.

B. Lower Bound of the Achievable Rate

In this paragraph we present a feasible schedule that bounds the achievable rate from below and show that the rate is attainable. We first describe the schedule and show that in each cycle a message from each node in the network is delivered to the gateway and the distance between any two transmitters is greater than or equal to F. We distinguish between odd and even F cases.

Schedule for odd F:

$$\begin{aligned}\{B_{(1,k)}(l+iF)\,|_{0\le i\le\lfloor (L-l)/F\rfloor}\\ B_{(1,k+1)}((i+1)F-l-2)\,|_{0\le i\le\lfloor (L-F+l+2)/F\rfloor}\\ B_{(1,k-1)}((i+1)F-1)\,|_{0\le i\le\lfloor (L-F+1)/F\rfloor}\ \text{if } l=0,\\ 1\le k\le m, 0\le l\le\lceil F/2\rceil-2\}\end{aligned} \quad (39)$$

with the following conventions: $k+1=1$ if $k=m$ and $k-1=m$ if $k=1$.

We schedule all groups for $i=0$ sequentially starting with $l=0$ cyclically going over all k and gradually increasing l until $l=\lceil\frac{F}{2}\rceil-2$. Groups that share the same k,l indices but differ in i are scheduled simultaneously. The number of message transmission time slots each node is assigned coincides with the number of messages it must forward such that it forward one message of its own and one for each of its tree successors, i.e., node $n_{(i,j)}$ receives $|B_{(i,j)}|$ messages. An example of optimal schedule is given in Fig. 11 for a ternary tree with depth $L=11$ and spatial reuse $F=5$.

Next we show that in this schedule, a message from each node arrives to the gateway. Since the number of time slots assigned to each node is exactly the number of messages the node has to forward, it is sufficient to show that all nodes are scheduled for transmission. The set $\{B_{(1,k)}(l)\,|\,1\le k\le m, 0\le l\le \lceil\frac{F}{2}\rceil-2\}$ covers all nodes in the first $\lceil\frac{F}{2}\rceil-1$ layers. The set $\{B_{(1,k+1)}((F-l)-2)\,|\,1\le k\le m0\le l\le\lceil\frac{F}{2}\rceil-2\}$ covers all nodes in layers $\lceil\frac{F}{2}\rceil,\dots,F-1$ and the set $\{B_{(1,k-1)}(F-1)\,|\,1\le k\le m\}$ covers the nodes in layer F; all nodes in the first F layers are covered. Since each set is scheduled to transmit with all sets that are iF layers away from it $0\le i\le\frac{L}{F}$, all nodes in the network are scheduled for transmission.

The set $\{B_{(1,k)}(l)\,|\ \le k\le m, 0\le l\le\lceil\frac{F}{2}\rceil-2\}$ which corresponds to the set S_{odd} is scheduled sequentially (no spatial reuse). Subgroups $\{B_{(1,k+1)}((F-l)-2), B_{(1,k-1)}(F-1)\,|\ \le k\le m, 0\le l\le\lceil\frac{F}{2}\rceil-2\}$ are scheduled simultaneously with $\{B_{(1,k)}(l)\}$, all belonging to different main branches of the tree; hence the distance between any two nodes belonging to different subgroups is equal to the sum of their respective distance to the gateway, and is greater than F hops. The rest of the nodes scheduled at the same time are kept $iF\ i\ge 1$ hops apart. Consequently, the distance between any two nodes transmitting simultaneously is greater than or equal to F. Therefore, spatial reuse is always satisfied.

The set of flows forwarded by $B_{(1,k)}(l+h)$ is only a subset of the flows forwarded by $B_{(1,k)}(l)$ hence, $|B_{(1,k)}(l)|>|B_{(1,k)}(l+h)|\ \forall h\ge 1$. Due to symmetry $|B_{(1,k)}(l)|=|B_{(1,k+e)}(l)|\ \forall e$, $(k+e) \bmod m$, which gives $|B_{(1,k)}(l)|>|B_{(1,k+l)}(l+h)|$. Therefore, the transmission duration of each set of flows scheduled simultaneously is determined by the transmission duration of the subset of flows that belongs to the lowest layer. Accordingly the transmission duration of the schedule complete cycle is determined by the transmission duration of set $\{B_{(1,k)}(l)|\le k\le m, 0\le l\le\lceil\frac{F}{2}\rceil-2\}$. Thus, the duration of the schedule is equal to the sum of $|B_{(1,k)}(l)|, 1\le k\le m, 0\le l\le\lceil\frac{F}{2}\rceil-2$ which is exactly K_{odd}; hence the schedule rate is $C(P)/K_{\text{odd}}$ as in (19). Since the upper and lower bounds coincide, the achievable rate is $C(P)/K_{\text{odd}}$.

For the even case, we use the schedule given in (39), adding the following additional group scheduled subsequently to the other groups:

$$B_{(1,k)}(F/2+iF)|_{1\le k\le m, 0\le i\le\lfloor(\frac{2L-F}{2F})\rfloor}. \quad (40)$$

All main branches $1\le k\le m$ in this group are scheduled simultaneously. Moreover the distance between the nodes that belong to different subgroups is at least F hops, hence they can transmit without interfering with each other. The rest of the proof follows the same procedure as the odd case where S_{even} replaces S_{odd}. An example of an optimal schedule is given in Fig. 12 for a ternary tree with depth $L=13$ and spatial reuse $F=6$.

Remark 1: The schedule for the binary tree is different from the other cases since for $k=1$, both indices $k+1$ and $k-1$ equal 2. Therefore, it corresponds to two subgroups in the

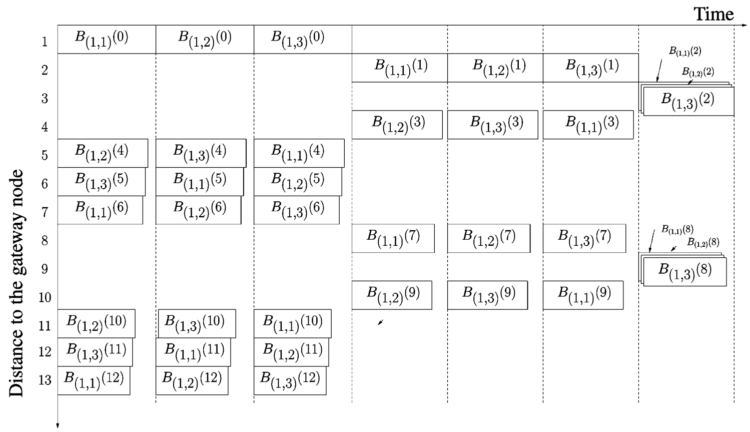

Fig. 12. An example of optimal schedule for ternary tree with even spatial reuse factor as determined in (39) and (40). The depth of the tree is $L = 13$ and the spatial reuse $F = 6$. In each block, we indicate the subset of messages $B_{(1,k)}(l)$ that are scheduled to transmit sequentially in arbitrary order. For the messages that belong to the same layer but to different subsets, e.g., $B_{(1,1)}(2)$,$B_{(1,2)}(2)$ and $B_{(1,3)}(2)$, we superimpose their corresponding boxes to show that they transmit simultaneously.

same main branch in (39) that interfere with each other and thus cannot be scheduled simultaneously. For example, in Fig. 11, $B_{(1,3)}(4)$ corresponds to $B_{(1,1)}(4)$ according to the schedule described by (39) and therefore cannot be scheduled simultaneously with $B_{(1,1)}(5)$. However, we show that a schedule which realizes the rate lower bound exists for the binary case. The main idea behind this result is to utilize spatial reuse inside the layers of a main subtree. Indeed, in order to achieve the lower bound, all messages $B_{(1,k)}(l), l = \lceil F/2 \rceil, \ldots, F$ have to be scheduled within the transmission duration of the messages $B_{(1,k)}(l), l = 1, \ldots, \lceil F/2 \rceil - 1$ for F odd and all messages $B_{(1,1)}(l), l = F/2+1, \ldots, F$ within the transmission duration of the messages $B_{(1,k)}(l), l = 1, \ldots, F/2-1$ for F even. Denote $F_{(k)}(l)$ as the spatial reuse factor within the nodes in layer l of the main branch k; due to the symmetry of the considered network topology, $F_{(1)}(l) = F_{(2)}(l), \forall l$. According to the spatial reuse constraint such that any receiving node has to be at least $F-1$ hops apart of an interfering node, it can be shown that $F_{(k)}(l) = m^{l-1}/2^{\max(0, l-\lceil F/2 \rceil)}$, i.e., for layers $1, \ldots, \lceil F/2 \rceil$, only one node among the nodes that belong to layer l in branch k can transmit at a time; in layer $\lceil F/2 \rceil + 1$, two nodes can transmit simultaneously; in layer $\lceil F/2 \rceil + 2$, four nodes can transmit simultaneously, etc. Therefore, the total duration to transmit the $|B_{(1,k)}(l)|$ messages becomes $(|B_{(1,k)}(l)|/2^{\max(0, l-\lceil F/2 \rceil)})$ time slots if spatial reuse inside the layers $iF, iF+1, i \geq 1$ is exploited. Since $|B_{(1,k)}(F-2)|$ and $|B_{(1,k)}(F-1)|$ are smaller than $|B_{(1,k)}(1)|$ and $2^{\max(0,F-\lceil F/2 \rceil)} > 2^{\max(0,F-1-\lceil F/2 \rceil)} \geq 2, B_{(1,2)}(F-2)$ and $B_{(1,2)}(F-1)$ (resp. $B_{(1,1)}(F-2)$ and $B_{(1,1)}(F-1)$) can be scheduled simultaneously with $B_{(1,1)}(0)$ (resp. $B_{(1,2)}(0)$). An example of optimal schedule is given in Fig. 13 for a binary tree with depth $L = 11$ and spatial reuse $F = 5$.

Remark 2: This approach is valid in the binary case with spatial reuse factor $F > 3$ (we assume throughout the paper that $F > 2$). For $F = 3$, it needs further modifications. If $F = 3$, the challenge is to schedule $B_{(1,2)}(F-2=1)$ and $B_{(1,2)}(F-1=2)$ simultaneously with $B_{(1,1)}(0)$. However, according to the spatial reuse inside layer, two nodes can be scheduled simultaneously inside layer 3 but not inside layer 2. In that case, even with spatial reuse inside layer 3, we have $|B_{(1,2)}(1)| + |B_{(1,2)}(2)|/2 > |B_{(1,1)}(0)|$ and $|B_{(1,1)}(1)| + |B_{(1,1)}(2)|/2 > |B_{(1,2)}(0)|$. However, a node that belongs to $B_{(1,k)}(1)$ can be scheduled with a node that belongs to $B_{(1,k)}(2), k = 1, 2$ according to the spatial reuse policy. This schedule realizes the bound if $\max(|B_{(1,1)}(1)|, |B_{(1,1)}(2)|) > |B_{(1,2)}(0)|$ and *vice versa*. It is verified since $B_{(1,k)}(0) > B_{(1,k)}(1) > B_{(1,k)}(2)$. An example of an optimal schedule is given in Fig. 14 for binary tree with depth $L = 6$ and spatial reuse is $F = 3$.

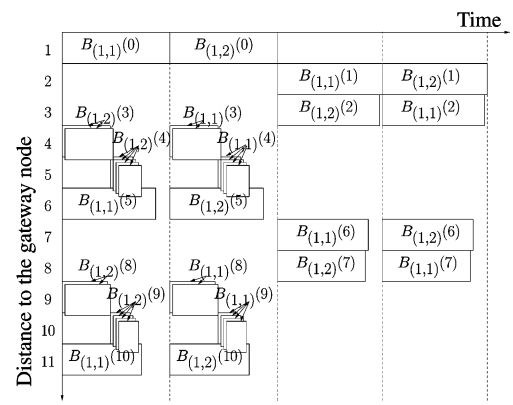

Fig. 13. An example of optimal schedule for binary tree as determined in Remark 1. The depth of the tree is $L = 11$ and the spatial reuse 5. For the sake of clarity, we use the same spatial reuse factor inside the layers 4 and $4 + F = 9$, and inside the layers 5 and $5 + F = 10$. In order to achieve (19), some messages inside subsets $B_{1,k}(l)$ have to be scheduled simultaneously. For example, two messages inside $B_{(1,2)}(3)$ and four messages inside $B_{(1,2)}(4)$ have to transmit simultaneously.

APPENDIX E
PROOF OF THEOREM 6

We extend the basic schedule structure suggested for the non cooperative case as presented by (39) and described in Fig. 11 to TRP. The main two differences between both schedules are 1) for spatial reuse factor F, the distance between two sources

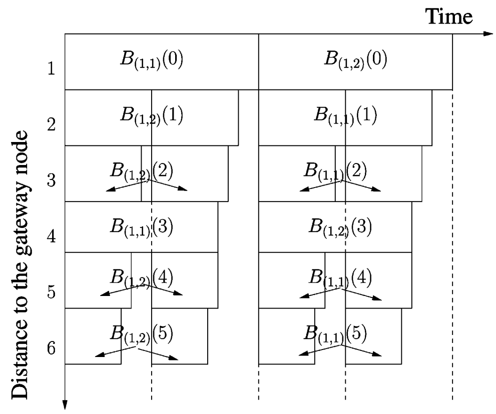

Fig. 14. An example of optimal schedule for binary tree with spatial reuse $F = 3$ as determined in Remark 2. The depth of the tree is $L = 6$.

transmitting simultaneously should be at least $F+1$ hops with TRP, e.g., The set of nodes S_{odd} that cannot transmit simultaneously in Fig. 10 corresponds now to a spatial reuse factor $F = 6$ instead of 7 in the noncooperative case. Similarly, the set S' corresponds to $F = 7$ with TRP instead of $F = 8$. The schedule in (39) should also be modified accordingly: The spatial reuse factor F is replaced by $F+1$ along the schedule and the schedule for even F corresponds to odd F in TRP and *vice versa*; 2) in the noncooperative case, we could schedule the nodes in each subgroup $B_{1,k}(l), \forall l$ sequentially and still realize the achievable rate. By scheduling the nodes that belong to each group sequentially, it is not necessarily true that the groups belonging in the first $F/2$ first layers dominate the cycle duration with TRP. We show next that these groups dominate the cycle duration only if we further exploit spatial reuse within the subgroups $B_{1,k}(l) l > \lceil \frac{F}{2} \rceil$.

Denote $T_{(1,k)}(l)$ as the duration to transmit all messages which belong to the set of nodes $B_{1,k}(l)$. In the noncooperative case, we had: $T_{(1,k)}(l) = |B_{1,k}(l)|$ time slots. In order to determine the duration of $T_{(1,k)}(l)$ for TRP, we first determine the transmission duration of any node in layer l. The transmission duration for a source node in layer $l+k$ to its upstream one hop neighbor in layer $l+k-1$ is equal to one time slot as it is in the non cooperative case. From the latter node to his neighbor in layer $l+k-2$, only $(1-\alpha)$ time slot is needed with TRP, α being defined in (12); from the latter node to his neighbor in layer $l+k-3$, only $(1-\alpha+\alpha^2)$ time slot is necessary, etc. Continuing the same reasoning, the transmission duration to transmit the necessary information of a node that belongs to layer $l+k$ from a node in layer l to its one-hop neighbor in layer $l-1$ with TRP is equal to $\sum_{i=0}^{k}(-\alpha)^i$. A node in layer l has m^k descendant nodes in layer $k+l$. Therefore, in order to transmit the information of all its descendant nodes in addition to its own information, a node in layer l requires $(\sum_{k=0}^{L-l} m^k \sum_{i=0}^{k}(-\alpha)^i)$ time slots. Hence, with TRP, $T_{(1,k)}(l)$ is equal to

$$T_{(1,k)}(l) = \left[\sum_{k=0}^{L-l} m^{k+l-1} \sum_{i=0}^{k}(-\alpha)^i\right].$$

The main challenge to determine an optimal schedule in the cooperative case comes from the fact that $T_{(1,k)}(l)$ might be greater than $T_{(1,k)}(l')$ with $l < l'$. Therefore, the use of spatial reuse within layer l is essential. The total duration to transmit the information of all nodes $B_{1,k}(l)$ becomes

$$T'_{(1,k)}(l) = \left[\sum_{k=0}^{L-l} m^{k+l-1} \sum_{i=0}^{k}(-\alpha)^i\right] \Big/ m^{\max(0,l-\lceil F/2 \rceil)}.$$

For odd F, we show that the schedule depicted in Fig. 12 is also valid for the cooperative case with $F = 5$. This is equivalent to showing that the transmission duration for the messages of the first $\lceil F/2 \rceil$ layers is dominant, i.e., $T'_{(1,k)}(0) \geq \max(T'_{(1,k)}(i(F+1)-2), T'_{(1,k)}(i(F+1)-1)), i \geq 1, T'_{(1,k)}(1) \geq T'_{(1,k)}(i(F+1)-3), i \geq 1, \ldots, T'_{(1,k)}(\lceil F/2 \rceil - 2) \geq T'_{(1,k)}(\lceil F/2 \rceil + i(F+1)), i \geq 0$.

Lemma 1: The total duration $T'_{(1,k)}(l)$ to transmit all messages $B_{(1,k)}(l)$ when full spatial reuse is exploited within layer l is greater than $T'_{(1,k)}(l'), l' > l$ as long as the number of nodes that can transmit simultaneously without interference in set $B_{(1,k)}(l')$ is at least m times greater than in set $B_{(1,k)}(l)$, i.e., $T_{(1,k)}(l) - T_{(1,k)}(l')/m \geq 0$.

$$\begin{aligned} &T_{(1,k)}(l) - T_{(1,k)}(l')/m \\ &\quad = \left[\sum_{k=0}^{L-l} m^{k+l-1} \sum_{i=0}^{k}(-\alpha)^i\right] \\ &\qquad - \left[\sum_{k=0}^{L-l} m^{k+l'-2} \sum_{i=0}^{k}(-\alpha)^i\right] \\ &\quad = \frac{1}{m^2(1+\alpha)(m-1)(1+\alpha m)} \cdot [m^{L+2}(m-1) \\ &\qquad \times (\alpha + (-\alpha)^{L+2-l}) + m^{L+1}(m-1) \\ &\qquad \times (1-(-\alpha)^{L+2-l'}) \\ &\qquad + (1+\alpha)(m^{l'} - m^{l+1})]. \end{aligned}$$

All terms in the numerator are positive for any $\alpha, 0 \leq \alpha \leq 1$ with $m > 1$ and $l' > l$ which completes the Proof of Lemma 1.

From Lemma 1, the schedule presented in Fig. 12 is also valid for the non cooperative case with F odd $(F = 5)$.

For F even, by applying Lemma 1, we have $T'_{(1,k)}(l) - T'_{(1,k)}(F-1-l) \geq 0, l = 1, \ldots F/2-1$. However, it can be shown that the quantity $T'_{(1,k)}(F/2-1) - T'_{(1,k)}(F/2)$ is not always positive even if all $T'_{(1,k')}(F/2)$ simultaneously transmit with $T'_{(1,k)}(F/2-1)$ with $k' = 1, \ldots, m, k' \neq k$. In the latter case, it can be negative only for marginal scenarios, i.e., very large values of SNR and for an even number of layers in the tree; for this reason, we omit the analysis of this extreme case in the paper. Otherwise, it is positive and it is possible to find a schedule which realizes the achievable rate (20). An example of optimal schedule is given in Fig. 15 for a ternary tree with spatial reuse $F = 4$. In order to realize the TRP achievable rate, we exploit the spatial reuse within the layers 3 and 4. Indeed, nodes that belong to $B_{(1,1)}(3), B_{(1,2)}(3)$ and $B_{(1,3)}(3)$ are scheduled to transmit simultaneously. In layer 4, higher spatial reuse can be used among the nodes. however, for the sake of simplicity, we also assume that three nodes within

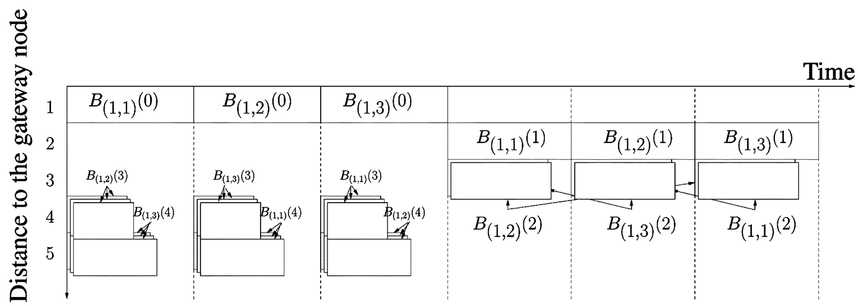

Fig. 15. An example of optimal schedule for ternary tree as determined in (20) with Turbo Relaying Protocol. The spatial reuse is $F = 4$. In order to achieve (20), spatial reuse inside subsets $B_{(1,k)}(l)$ needs to be exploited; e.g., three messages that belong to $B_{(1,2)}(3)$ are simultaneously scheduled. Additionally, messages that belong to the same layer but different subsets $B_{(1,k)}(l)$ have to be scheduled simultaneously, e.g., half of the messages in $B_{(1,2)}(2)$ are scheduled with half of the messages in $B_{(1,3)}(2)$; half of the messages in $B_{(1,3)}(2)$ are scheduled with half of the messages in $B_{(1,1)}(2)$; and half of the messages in $B_{(1,1)}(2)$ are scheduled with half of the messages in $B_{(1,2)}(2)$.

layer 4 transmit simultaneously. According to Lemma 1, it is sufficient to guarantee that the transmission duration of nodes $B_{(1,1)}(0), B_{(1,2)}(0)$ and $B_{(1,3)}(0)$ dominates the transmission duration of nodes $B_{(1,i)}(3)$ and $B_{(1,i)}(4), i = 1, 2, 3$ in order to achieve (20).

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