Cross-Layer Optimal Spectrum Sensing Duration and Scheduling in Cognitive Networks

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ABSTRACT

We explore the effects of spectrum sensing time parameters on non-cooperative dynamic spectrum access by jointly considering the PHY layer sensing and the MAC layer sensing scheduling. Our work not only incorporates singular sensing performance measures in terms of the probability of missed detection and false alarm, but also takes into account the likeliness of missed detection of busy-idle channel state transitions between consecutive sensing executions. In this paper, we develop two metrics to quantify the spectrum sensing performance of a singular node, namely the *spectrum efficiency* for detecting available spectrum and the *sensing robustness* for avoiding interference to primary users, and present a general framework for deriving the desired results, which accommodates various primary channel occupancy patterns.

Categories and Subject Descriptors

C.2.3 [**Network Operation**]: Network monitoring

General Terms

Algorithms, Design, Performance

Keywords

Cognitive radio, Spectrum sensing, Sensing time

1. INTRODUCTION

Spectrum scarcity has become a major obstacle for a rapid proliferation of new wireless technologies and systems. Recent studies in spectrum usage have revealed that this scarcity is not only a symptom of the increased use of the spectrum, but also result of inherent inefficiencies in the spectrum assignment policies [1–3]. Dynamic spectrum access (DSA) has been proposed as a promising method to mitigate shortcomings of the current spectrum regime by allowing unlicensed operations in temporarily or spatially underutilized regions of the RF spectrum, the so-called "spectrum white spaces". DSA can be enabled without primary user sensing by using white space databases. However, different sensing mechanisms could be used to significantly enhance efficiency of cognitive radios (CR) that exploit white spaces. The spectrum sensing based systems could operate independently or more likely would be used in cooperation with databases.

In this paper we study the optimality of spectrum sensing parameters in the context of DSA. The spectrum sensing process is distributed among the lower layers of the protocol stack. At the PHY layer, the analysis of channel samples to detect the presence of primary signals is conducted by using advanced detection techniques. Results of this sensing process are reported to the MAC layer that controls the communication of the radio system and is in charge of scheduling transmission and sensing execution. In this paper we are interested in understanding and optimizing design issues associated with the sensing scheduling including how long, how often and which regions of the spectrum should be sensed. Specifically we aim to optimize scheduling of transmission and sensing.

In essence we seek to find optimal values for *spectrum sensing duration, T_s,* and *sensing request interval, T_p.* Although only two major parameters are considered, the tradeoffs and interplay between these two quantities are quite complex. The tradeoffs between sensing duration and intervals are well known. Too long sensing duration or too short interval between two sensing executions lowers system efficiency and increases energy consumption, whereas too short sensing duration or too long interval would lead to increased probability to fail to detect primary transmissions. Since the performance of any sensing scheduling scheme can be evaluated only in the context of the channel use pattern (of primary users), we will employ a renewal process based primary activity model as used in [4]. We have proposed two efficiency metrics which allow us to gain a better understanding of the results of the balancing process: the *spectrum efficiency* metric describing a node's ability to detect vacant spectrum, *sensing robustness* on the other hand giving a measure for a node's capability to detect a primary transmission and avoid harmful interference to primary users. In the paper, we formulate the sensing scheduling of T_s and T_p as an optimization problem to maximize these two metrics based on a general alternating renewal channel occupancy model.

The rest of the paper is organized as follows. In Section 2, we summarize the related work. Section 3 describes the system model and assumptions. We analyze the effects of MAC

layer sensing scheduling and present a general framework to compute two metrics in Section 4. In Section 5, we derive the model expressions for the case of exponential distribution of primary traffic and conduct the simulation and numerical results analysis. The paper concludes with Section 6.

2. RELATED WORK

There has been a number of publications on selecting optimal spectrum sensing durations T_s [5–9] and periods T_p [8–13]. Regarding the sensing duration, Liang et al. in [5] optimize T_s by studying the tradeoff between the sensing time overhead T_s and the data transmission time T_d ($T_d = T_p - T_s$) of secondary users over a single channel without violating the target probability of detection. The work is further extended in [6] and [8] for wideband channel and p-persistent CSMA MAC access scenarios, respectively. Moreover, Guo et al. in [7] derive the optimal T_s by minimizing the available channel searching time in a sequential channel search manner. However, all these models consider only the sensing imperfections caused by the PHY layer sensing such as missed detection and false alarm. Another important sensing performance criterion, i.e., missed detection of busy-idle channel state transitions between two consecutive spectrum measurements due to the MAC layer sensing scheduling, has not been taken into account in these calculations.

In [9–13], the authors take the MAC sensing scheduling as a basis for the design of sensing period T_p. However, in [10], [11] and [13], only one aspect of undiscovered busy-idle transitions during T_d is modeled. Authors in [10] and [13] investigate channel status transitions from busy to idle to derive the undiscovered idle durations, while Pei et al. in [11] focus on transitions from idle to busy to minimize the undiscovered busy durations. In [9] and [12], both these two cases are considered in order to find optimum sensing timing parameters. The authors in [12], however, focus on finding only the optimal T_p regardless of T_s. The problem in [9] is formulated to maximize T_d/T_p satisfying certain interference constraints. This is not accurate since a lengthy T_d does not mean more spectrum opportunities within one T_p. There exists a possibility that a large fraction of spectrum opportunities cannot be detected within T_d.

In addition, we note that most of the existing relevant papers, such as [8, 9, 11, 12], directly employ an exponential busy-idle channel occupancy model in the analysis, and the busy-idle durations within each period are assumed to follow the same exponential distribution. Different from them, [13] analyzes a general alternating renewal distribution of channel occupancy. In this paper, we further extend the work presented in [13] to provide more comprehensive and accurate models by thoroughly modeling the MAC-PHY cross layer sensing behaviors. T_s and T_d are jointly designed instead of being modeled isolatedly as in [13].

3. SYSTEM MODEL PRELIMINARIES

We consider a one-channel primary network where a single secondary user performs local sensing and accesses the channel based on the detected vacancy. The sensing consists of the MAC layer scheduling and the PHY layer spectrum measurement. We assume the MAC layer performs a periodic sensing with the interval T_p. At the PHY layer, the secondary user collects samples of the channel and processes

them for an indication of the primary system activity during T_s. The duration between two consecutive spectrum measurements is dedicated for data transmissions and denoted by T_d ($T_d = T_p - T_s$).

The length of T_s varies with different PHY layer sensing techniques and determines the achievable probabilities of detection P_d and false alarm P_f, where the former denotes the probability of a PHY layer technique to correctly detect the presence of a primary signal, and the latter denotes the false alarm probability for a primary signal detection. In our work we adopt the energy detector model proposed in [11] where the primary signals are assumed to be i.i.d distributed and complex PSK modulated with zero mean. This is done without losing generality of our approach. The noise at secondary receivers is circularly symmetric complex Gaussian with zero mean and variance of σ^2, and is independent from primary signals. With this model, the probabilities of detection and false alarm are expressed as functions of T_s as follows:

$$P_d = Q\left(\left(\frac{\epsilon}{\sigma^2} - \gamma - 1\right)\sqrt{\frac{T_s f_s}{2\gamma + 1}}\right), \quad (1)$$

$$P_f = Q\left(\left(\frac{\epsilon}{\sigma^2} - 1\right)\sqrt{T_s f_s}\right), \quad (2)$$

where $Q(\cdot)$ is the complementary distribution function of a standard Gaussian variable and is given by $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-t^2/2}dt$, γ and ϵ are the received signal-to-noise (SNR) ratio measured at the secondary receiver and the energy detection threshold respectively. A primary signal is detected when $\gamma > \epsilon$. f_s is the signal sampling rate.

In this paper, primary traffic activity over a licensed channel is modeled as an alternating renewal process consisting of busy and idle periods which are i.i.d distributed. We use P_{on} and P_{off} to denote the probabilities of ON and OFF states of a channel respectively, and $P_{on} + P_{off} = 1$. Here ON and OFF denote the channel busy and idle states, respectively.

4. OPTIMAL SENSING TIMING DESIGN

4.1 Problem Formulation

A good spectrum sensing scheme allows the secondary system to reliably detect white spaces and protect licensed users from detrimental interference. In our analysis, we use two key metrics to characterize the performance of a sensing scheme, i.e., the *sensing efficiency* on detecting the available spectrum and the *sensing robustness* for avoiding interference to the primary networks.
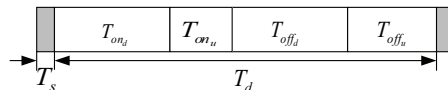

Figure 1: A data transmission duration T_d consisting of four time components

As shown in Figure 1, one data transmission period T_d between consecutive channel sensing measurements can be decomposed into four distinct components: discovered primary transmission duration T_{on_d} and channel idle duration T_{off_d}, undiscovered primary transmission duration T_{on_u} and

channel idle duration T_{off_u}. It should be noted that some of these time components may occur non-continuously or even not appear within T_d. We use notations T_{on_d}, T_{off_d}, T_{on_u} and T_{off_u} to denote the total amount of durations for each type of time components during one T_d. Given a portion of spectrum, the real usable spectrum depends on the secondary users' capability to discover the duration of spectrum availability, i.e., to increase T_{off_d} and decrease T_{off_u}. A high spectrum *sensing efficiency* requires a long T_{off_d} with a short T_s. We use the normalized discovered spectrum opportunity to describe the first sensing performance metric as:

$$\eta = \frac{E(T_{off_d})}{T_s + T_d}, \qquad (3)$$

where $E(\cdot)$ is the expectation function, and T_s and T_d are assumed to be fixed. It should be noted that secondary users will initiate transmissions during both T_{off_d} and T_{on_u}. However, since numerous measurement campaigns have shown that primary activity T_{on} is small in reality (say, less than 30%), and the secondary transmission could also be corrupted by primary users during T_{on_u}, it is reasonable to consider T_{off_d} to be a dominant factor characterizing the secondary achievable bandwidth utilization over a primary channel.

The second spectrum sensing performance metric is considered in terms of protecting the primary system from interference, and is determined by the length of T_{on_u} during which secondary users are not aware of the existence of primary signals. The transmission power of secondary users could result in a primary outage event during this period. Similar to (3), we use the normalized interference duration to characterize the *sensing robustness* as follows:

$$\zeta = \frac{E(T_{on_u})}{E(T_{on})}, \qquad (4)$$

where $E(T_{on}) = E(T_{on_u}) + E(T_{on_d})$.

The objective of spectrum sensing is to achieve accurate detection results by minimizing the undiscovered fraction of the spectrum states. As a result, it is expected that the achievable throughput can be maximized while licensed communication is sufficiently protected. In formula, the sensing time parameter decision in our work can be expressed as an optimization problem as:

$$\{T_s^*, T_d^*\} = \arg\max_{T_s, T_d} \eta \bigcap \arg\min_{T_s, T_d} \zeta, \qquad (5)$$

where T_s^* and T_d^* are the optimal sensing duration and data transmission duration respectively. It should be noted that η and ζ might not find one common set of T_s^* and T_d^* for their extrema. These two metrics have to be compromised to reach certain sensing performance quality. In practical scenarios, an interference tolerance threshold is usually predefined to protect the communication of primary users. Hence, the optimal problem can be further described as:

$$\{T_s^*, T_d^*\} = \arg\max_{T_s, T_d} \eta, \text{subject to } \zeta(T_s^*, T_d^*) \leq \zeta_{th}, \qquad (6)$$

where ζ_{th} is the predefined threshold with which the primary users are defined as being sufficiently protected along the temporal dimension. In the following subsection, we present the approach to derive (3) and (4).
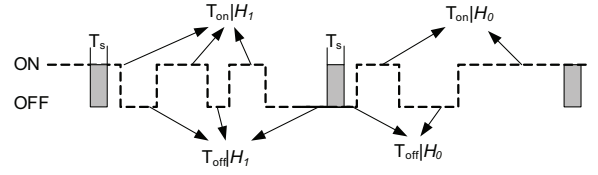

Figure 2: $T_{on}|H_1$ **and** $T_{off}|H_1$ **stand for the total amount of ON and OFF durations in one period respectively when the channel state in** T_s **is ON.** $T_{on}|H_0$ **and** $T_{off}|H_0$ **stand for the total amount of ON and OFF durations in one period respectively when the channel state in** T_s **is OFF.**

4.2 Analytical model

Observing (3) and (4), the computation of η and ζ can be transformed to derive the unknown time variables T_{on_u}, T_{on_d} and T_{off_u}. Figures 2 illustrates the channel ON-OFF state transitions in a sensing period which start with busy or idle channel state. It is obvious that, with the periodic sensing, secondary users are unable to detect the ON-OFF channel state transitions between two consecutive spectrum measurements, which results in the same detrimental effects caused by PHY layer miss detection and false alarm. This phenomenon must be taken into account when ON-OFF state transitions in a primary channel take place frequently. Let H_0 and H_1 denote the channel states OFF and ON during T_s. We assume that the duration T_s is short enough and does not contain any state transition in itself. The discovery of $T_{on}|H_1$, $T_{off}|H_1$, $T_{on}|H_0$ and $T_{off}|H_0$ depends on how the state during T_s is correctly or incorrectly detected at the PHY layer. We have:

$$E(T_{on_d}) = P_{on}P_d E(T_{on}|H_1) + P_{off}P_f E(T_{on}|H_0), \qquad (7)$$

$$E(T_{on_u}) = P_{on}(1 - P_d)E(T_{on}|H_1) + P_{off}(1 - P_f)E(T_{on}|H_0), \qquad (8)$$

$$E(T_{off_d}) = P_{on}(1 - P_d)E(T_{off}|H_1) + P_{off}(1 - P_f)E(T_{off}|H_0), \qquad (9)$$

$$E(T_{off_u}) = P_{on}P_d E(T_{off}|H_1) + P_{off}P_f E(T_{off}|H_0). \qquad (10)$$

We compute $E(T_{on}|H_1)$, $E(T_{off}|H_1)$, $E(T_{on}|H_0)$ and $E(T_{off}|H_0)$ based on the general model developed in [13]. Let $F_{on}(t)$ and $F_{off}(t)$ denote the c.d.f. of the ON and OFF periods respectively. Similarly, $f_{on}(t)$ and $f_{off}(t)$ are their p.d.f.s. In [13], the Laplace transforms of $E(T_{off}|H_1)$ and $E(T_{off}|H_0)$ are derived as:

$$E(T_{off}|H_1)^*(s) = \frac{G_{on}^*(s)}{E(T_{on})s^2} \cdot \frac{f_{off}^*(0) - f_{off}^*(s)}{1 - f_{off}^*(s)f_{on}^*(s)}, \qquad (11)$$

$$E(T_{off}|H_0)^*(s) = \frac{1}{E(T_{off})s^2}[G_{off}^*(0) - G_{off}^*(s) \cdot \frac{1 - f_{off}^*(0)f_{on}^*(s)}{1 - f_{off}^*(s)f_{on}^*(s)}], \qquad (12)$$

where $(\cdot)^*$ denotes the Laplace transform of function $(\cdot)$; $G_{on}(t) = 1 - F_{on}(t)$ and $G_{off}(t) = 1 - F_{off}(t)$. Based on the same theory, we derive the Laplace transforms of $E(T_{on}|H_1)$

and $E(T_{on}|H_0)$ as follows:

$$E(T_{on}|H_1)^*(s) = \frac{1}{E(T_{on})s^2}[G^*_{on}(0)-$$

$$G^*_{on}(s) \cdot \frac{1 - f^*_{on}(0)f^*_{off}(s)}{1 - f^*_{on}(s)f^*_{off}(s)}], \quad (13)$$

$$E(T_{on}|H_0)^*(s) = \frac{G^*_{off}(s)}{E(T_{off})s^2} \cdot \frac{f^*_{on}(0) - f^*_{on}(s)}{1 - f^*_{on}(s)f^*_{off}(s)}. \quad (14)$$

The model can be applied directly to any alternating renewal processes where the p.d.f. and c.d.f. distributions of ON and OFF periods have closed form expressions of Laplace transforms, such as exponential and Erlang distributions.

5. ANALYSIS FOR EXPONENTIAL CHANNEL OCCUPANCY DISTRIBUTION

Here we analyze and derive optimal parameter selection in the case of the exponential distribution of ON-OFF periods of primary users. The exponential distribution for primary ON-OFF traffic is described with parameters β and α. Hence, the probability density functions are $f_{on} = \beta \exp(-\beta t)$ and $f_{off} = \alpha \exp(-\alpha t)$, respectively. The stationary distributions of the idle and busy periods are $P_{on} = \frac{\alpha}{\alpha+\beta}$ and $P_{off} = \frac{\beta}{\alpha+\beta}$, respectively. We carry out the inverse Laplace transform of (11)-(14) and obtain:

$$E(T_{off}|H_1) = \beta \cdot A(T_d), \quad (15)$$

$$E(T_{off}|H_0) = T_d - \alpha \cdot A(T_d), \quad (16)$$

$$E(T_{on}|H_1) = T_d - \beta \cdot A(T_d), \quad (17)$$

$$E(T_{on}|H_0) = \alpha \cdot A(T_d), \quad (18)$$

where $A(T_d) = \frac{1}{(\alpha+\beta)^2} \cdot [(\alpha + \beta) \cdot T_d - 1 + e^{-(\alpha+\beta)\cdot T_d}]$.

We use MATLAB to validate numerically the results for the exponential case. The primary channel is assumed to have a bandwidth of 6MHz. The sampling rate of sensing at the PHY layer is twice the bandwidth. We set the target values of $P_d = 0.9$ and $P_f = 0.1$ with $\gamma = -15$dB, and derive a fixed value of $\frac{\epsilon}{\sigma^2}$ which is further used in (1) and (2) to compute the P_d and P_f used in this section. We have run extensive computations and simulations with various combinations of α, β, T_d and T_s. The ratio of α/β is varied in the interval $[1/10, 1/9, \cdots, 1/2, 1, 2, \cdots, 10]$ and shows different spectrum occupancy statistics with $\beta \in \{0.01, 0.1, 1, 10, 100\}$. T_d is selected as a value of $w \times 1/\beta$, where w varies from $1/100$ to 100. Due to the page limit, we only show the most representative and relevant results in this paper.

5.1 Simulation results validation

In Figure 3, we compare the simulated and analytical results of ζ and η. It is observed that, for different T_d, both methods exhibit the same trend. The simulation results converge towards the analytical calculations for $T_d \gg T_s$. When T_d approaches T_s or less than T_s, the analytical results deviate from the simulation results, and the deviation increases as T_s becomes large. This discrepancy stems from our analysis that we assume that the duration T_s is very short compared to T_d and does not contain any state transition within itself. When T_s approaches T_d, the model becomes inaccurate since the exponential distribution of channel states in

T_d may no longer apply. Despite of this imprecision, we observed that the analytical performance is slightly better than the simulated results, which allows us to use the analytical results as upper bound guidelines in the design of sensing time parameters.
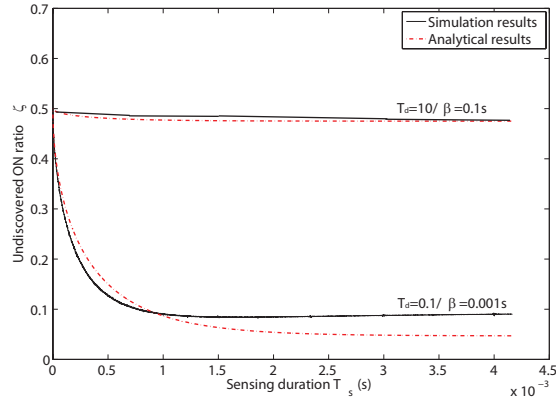
In the figure, it is observed that, for the case of $T_d = 0.001s$, the two performance metrics vary considerably as T_s changes. ζ initially decreases with the increase of the sensing duration T_s. When T_s reaches a particular threshold, e.g., 1ms in this simulation, ζ approximates a steady state regardless of T_s. For the normalized bandwidth utilization η, it firstly increases due to better sensing results with longer T_s. When T_s exceeds 0.2ms, η gradually decreases due to heavy sensing overhead T_s. This result also proves that the condition $T_d > T_s$ should be met in the sensing parameter selection. In contrast, for the case of $T_d = 0.1s$ (i.e., $T_d \gg T_s$), the two performance metrics become insensitive to the value of T_s. T_d becomes a predominant parameter in determining the system performance. However, our extensive simulation and analytical results advocate that we should avoid selecting time parameters that satisfy $T_d \gg T_s$, as this condition results in a large value of ζ which means a significant interference to primary systems. As an example we show $\zeta = 0.5$ for $T_d = 0.1s$ in Figure 3(a).

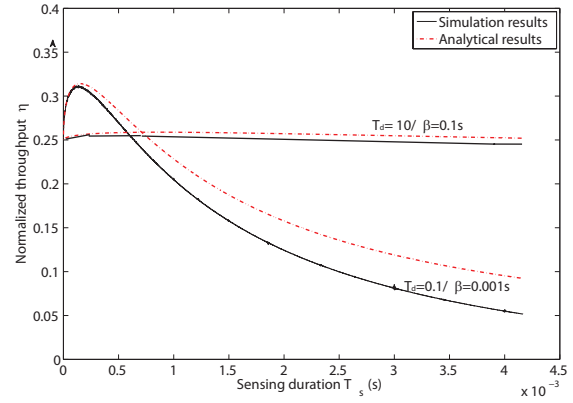### 5.2 Sensing time parameters T_s^* and T_d^* design

Figure 4 shows the analytical undiscovered ON duration ratio ζ and normalized bandwidth utilization η as a function of T_d, T_s, α, and β, respectively. It can be seen that $\zeta \le 0.1$ can be satisfied only when T_d is chosen as $T_d \le 0.2/\beta$, where $0.2/\beta$ is an approximate threshold. Our results show that this threshold applies to any values of α and β given that channel occupancy probability satisfies $P_{on} < 75\%$. This property is very useful in practice, because almost all the current measurement campaigns indicate that such level of temporal under-utilization is common in many frequency bands. In Figure 4(b), it can be observed that η firstly increases as T_d becomes large. When T_d exceeds a certain threshold (say, $0.2/\beta$), η decreases with T_d. This trend indicates that there always exists an optimal sensing period T_d^* which maximizes the bandwidth utilization. Let's use an example to show how to select T_s^* and T_d^* based on the results in the figure. Given a primary system interference threshold $\zeta_{th} = 0.1$, Figure 4(a) suggests that there is a range of T_s and T_d to satisfy this interference condition, e.g., $\{T_d^*, T_s^*\} \subset \{T_d = 0.2/\beta, T_s \ge 2 \times 10^{-3}\}$. In Figure 4(b), it is further observed that the maximize throughput can be achieved with $\{T_d^* = 0.2/\beta, T_s^* = 2 \times 10^{-3}\}$ seconds.

6. CONCLUSIONS

In this paper, we have developed a general and comprehensive model for the secondary user sensing timing parameter design that takes into account both the MAC and PHY cross-layer sensing effects. We have studied a baseline scenario where one secondary user accesses one channel based on its own detection behavior. The model allows a secondary user to achieve optimal bandwidth utilization over certain interference constraints. Our results show that for an exponential spectrum occupancy distribution with an average channel occupancy probability of less than 75%, the optimal data transmission duration should be chosen as a value no more than $0.2/\beta$, with which the missed incumbent detection ratio can be dropped below 10%. Correspondingly,
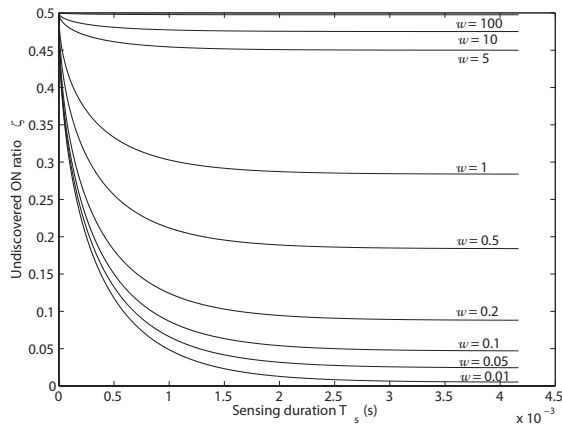
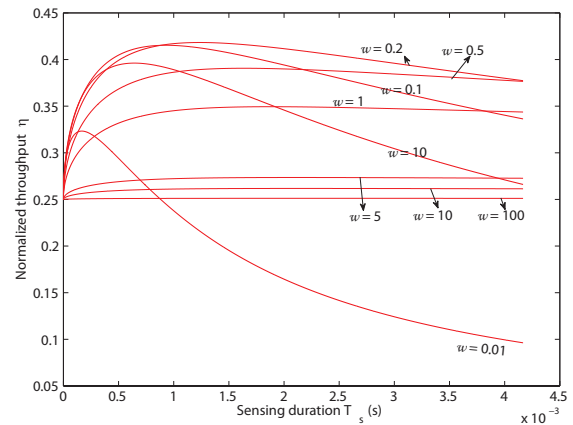(a) Undiscovered ON duration ratio ζ

(b) Normalized bandwidth utilization η

Figure 3: Comparison of simulated and analytical results ($\alpha/\beta = 1$, $\beta = 100/\mathbf{s}$).

(a) Undiscovered ON duration ratio ζ

(b) Normalized bandwidth utilization η

Figure 4: Analytical results with different T_s and $T_d = w/\beta$ ($\alpha/\beta = 1$, $\beta = 10/\mathbf{s}$).

there always exists an optimal T_s to maximize the bandwidth utilization. Although we have considered here only exponential primary user traffic patterns, the method is general. For some general distribution functions without closed form expression of Laplace transform, such as Pareto and log-normal, numerical evaluation can be done straightforwardly. A future work is required to extend model to the scenarios with multiple users cooperatively sensing and accessing a large number of channels.

Acknowledgment

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