Wavelet Packet Modulation for Wireless Communications

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***Abstract*— As proven by the success of OFDM, multicarrier modulation has been recognized as an efficient solution for wireless communications. Waveform bases other than sine functions could similarly be used for multicarrier systems in order to provide an alternative to OFDM.**

In this article, we study the performance of wavelet packet transform modulation (WPM) for transmission over wireless channels. This scheme is shown to be overall quite similar to OFDM, but with some interesting additional features and improved characteristics. A detailed analysis of the system's implementation complexity as well as an evaluation of the influence of implementation-related impairments are also reported.

***Index Terms*— Wavelet packet modulation, multicarrier modulation, orthogonal waveform bases**

I. Introduction

ATHOUGH the principle of multicarrier modulation is not recent, its actual use in commercial systems had been delayed until the technology required to implement it became available at reasonable cost [1]. Similarly, the idea of using more advanced transform than Fourier's as the core of a multicarrier system has been introduced more than a decade ago [2]. However, such alternative methods have not been foreseen as of major interest and therefore have received little attention. With the current demand for high performance in wireless communication systems, we are entitled to wonder about the possible improvement that wavelet-based modulation could exhibit compared to OFDM systems. We address this question in this article by taking theoretical as well as implementation aspects into account.

Several objectives motivate the current research on WPM. First of all, the characteristics of a multicarrier modulated signal are directly dependent on the set of waveforms of which it makes use. Hence, the sensitivity to multipath channel distortion, synchronization error or non-linear amplifiers might present better values than a corresponding OFDM signal. Little interest has been given to the evaluation of those system level characteristics in the case of WPM.

Moreover, the major advantage of WPM is its flexibility. This feature makes it eminently suitable for future generation of communication systems. With the ever-increasing need for enhanced performance, communication systems can no longer be designed for average performance while assuming channel conditions. Instead, new generation systems have to be designed to dynamically take advantage of the instantaneous propagation conditions. This situation has led to the study of flexible and reconfigurable systems capable of optimizing performance according to the current channel response [3]. A tremendous amount of work has been done recently to fulfill this requirement at the physical layer of communication systems: complex equalization schemes, dynamic bit-loading and power control that can be used to dynamically improve system performance. While WPM can take advantage of all those advanced functionalities designed for multicarrier systems, we show that it benefits also from an inherent flexibility. This feature together with a modular implementation complexity make WPM a potential candidate for building highly flexible modulation schemes.

Wavelet theory has been foreseen by several authors as a good platform on which to build multicarrier waveform bases [4], [5], [6]. The dyadic division of the bandwidth, though being the key point for compression techniques, is not well suited for multicarrier communication [7]. Wavelet packet bases therefore appear to be a more logical choice for building orthogonal waveform sets usable in communication. In their review on the use of orthogonal transmultiplexers in communications [2], Akansu *et al.* emphasize the relation between filter banks and transmultiplexer theory and predict that WPM has a role to play in future communication systems.

Lindsey and Dill were among the first to propose wavelet packet modulation. The theoretical fondation of this orthogonal multicarrier modulation technique and its interesting possibility of leading to an arbitrary time-frequency plane tiling are underlined [8]. Lindsey has further completed the work in this area by presenting additional results [9]. In particular, it is shown in this last paper that power spectral density and bandwidth efficiency are equal for WPM and standard QAM modulation. Moreover, WPM is placed into a multitone communication framework including alternative orthogonal bases such as M-band wavelet modulation (MWM) and multiscale modulation (MSM).

Additional research work on more realistic models of WPM-based transceivers has also been carried out. Maximum likelihood decoding for wavelet packet modulation has been addressed by Suzuki *et al.* [10]. Simulations for both flat fading and multipath channels have been performed for a receiver using a channel impulse response estimator, an MLSE and Viterbi decoder. It is shown that the use of wavelet packet specific characteristics leads to improved results. The study of an equalization scheme suited for WPM has also been done by Gracias and Reddy [11]. In addition, the multiresolution structure of the wavelet packet waveforms have shown to offer opportunity for improved synchronization algorithms [12], [13]. This has been exploited for instance in the case of WPM by Luise *et al.* [14]. All together, the research work provides insight on the theoretical performance of an actually implemented WPM-based modem, thus contributing to complete the research work required to lead to a fully implementable WPM-based communication system.

We focus in this article on reviewing the advantages of wavelet packet modulation in wireless communications. Theoretical background on this modulation scheme is first recalled. Specific issues of using such sets of waveforms for multicarrier communication systems are underlined, and an exhaustive comparison with OFDM is made. Special emphasis on the flexibility of this scheme is given. Section III reports the performance results of WPM in several typical wireless channel models. The case of multipath channels is detailed, and the performance versus complexity of channel equalization for such channels is studied. Results obtained are compared with those obtained with a classical OFDM scheme, since it is currently the reference in multicarrier systems. Section IV focuses on the effect of interference and implementation impairments. Then, the implementation complexity of a WPM system is also reported, since this is a key issue in wireless communication systems. We conclude by underlining the communications area where WPM appears to be a promising technology for future generation transceivers.

II. Communication system architecture

The simplified block diagram of the multicarrier communication system studied in this article is shown in Figure 1. The transmitted signal in the discrete domain, $x[k]$, is composed of successive modulated symbols, each of which is constructed as the sum of M waveforms $\varphi_m[k]$ individually amplitude modulated. It can be expressed in the discrete domain as:

$$x[k] = \sum_{s} \sum_{m=0}^{M-1} a_{s,m}\, \varphi_m\big[k - s\,M\big] \qquad (1)$$

where $a_{s,m}$ is a constellation encoded s-th data symbol modulating the m-th waveform. Denoting T the sampling period, the interval $[0, LT-1]$ is the only period where $\varphi_m[k]$ is non-null for any $m \in \{0..M-1\}$. In an AWGN channel, the lowest probability of erroneous symbol decision is achieved if the waveforms $\varphi_m[k]$ are mutually orthogonal, i.e.

$$\langle \varphi_m[k],\, \varphi_n[k] \rangle = \delta[m-n], \qquad (2)$$

where $\langle \cdot, \cdot \rangle$ represents a convolution operation and $\delta[i] = 1$ if $i = 0$, and 0 otherwise.

In OFDM, the discrete functions $\varphi_m[k]$ are the well-known M complex basis functions $w[t]exp\big(j2\pi \frac{m}{M}\, kT\big)$ limited in the time domain by the window function $w[t]$. The corresponding sine-shaped waveforms are equally spaced in the frequency domain, each having a bandwidth of $\frac{2\pi}{M}$ and are usually grouped in pairs of similar central frequency and modulated by a complex QAM encoded symbol. In WPM, the subcarrier waveforms are obtained through the WPT. Exactly as for ODFM, the *inverse* transform is used to build the transmitted symbol while the *forward* one allows retrieving the data symbol transmitted. Since wavelet theory has part of its origin in filter bank theory [15], the processing of a signal through WPT is usually referred as decomposition (i.e. into wavelet packet coefficients), while the reverse operation is called reconstruction (i.e. from wavelet packet coefficients) or synthesis.

In this paper, we limit our analysis to WPT that can be defined through a set of FIR filters. Though it would be possible to use other wavelets as well, those cannot be implemented by Mallat's fast algorithm [16] and hence their high complexity make them ill-suited for mobile communication. The synthesis discrete wavelet packet transform

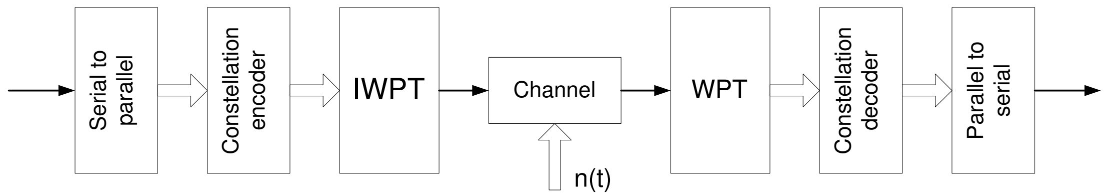

Fig. 1. Wavelet packet modulation functional block diagram

constructs a signal as the sum of $M = 2^J$ waveforms. Those waveforms can be built by J successive iterations each consisting of filtering and upsampling operations. Noting $\langle \cdot, \cdot \rangle$ the convolution operation, the algorithm can be written as:

$$\begin{cases} \varphi_{j,2m}[k] & = \left\langle h_{lo}^{rec}[k],\, \varphi_{j-1,m}[k/2] \right\rangle \\ \varphi_{j,2m+1}[k] & = \left\langle h_{hi}^{rec}[k],\, \varphi_{j-1,m}[k/2] \right\rangle \end{cases}$$
$$\text{with } \varphi_{0,m}[k] = \begin{cases} 1 & \text{for } k = 1 \\ 0 & \text{otherwise} \end{cases} \quad \forall m$$

where j is the iteration index, $1 \leq j \leq J$ and m the waveform index $0 \leq m \leq M-1$. Using usual notation in discrete signal processing, $\varphi_{j,m}[k/2]$ denotes the upsampled-by-two version of $\varphi_{j,m}[k]$. For the decomposition, the reverse operations are performed, leading to the complementary set of elementary blocks constituting the wavelet packet transform depicted in Figure 2. In orthogonal wavelet systems, the scaling filter h_{lo}^{rec} and dilatation filter h_{hi}^{rec} form a quadrature mirror filter pair. Hence knowledge of the scaling filter and wavelet tree depth is sufficient to design the wavelet transform [15]. It is also interesting to notice that for orthogonal WPT, the inverse transform (analysis) makes use of waveforms that are time-reversed versions of the forward ones. In communication theory, this is equivalent to using a matched filter to detect the original transmitted waveform.

A particularity of the waveforms constructed through the WPT is that they are longer than the transform size. Hence, WPM belongs to the family of overlapped transforms, the beginning of a new symbol being transmitted before the previous one(s) ends. The waveforms being M-shift orthogonal, the inter-symbol orthogonality is maintained despite this overlap of consecutive symbols. This allows taking advantage of increased frequency domain localization provided by longer waveforms while avoiding system capacity loss that normally results from time domain spreading. The waveforms length can be derived from a deteilled analysis of the tree algorithm [16, Section 8.1.1]. Explicitly, the wavelet filter of length L_0 generates M waveforms of length

$$L = (M-1)(L_0-1)+1. \tag{3}$$

In Daubechie's wavelet family [17] for instance, the length L_o is equal to twice the wavelet vanishing order N. For the order 2 Daubechie wavelet, L is equal to 4, and thus a 32 subcarrier WPT is composed of waveforms of length 94. This is therefore about three times longer than the corresponding OFDM symbol, assuming no cyclic prefix is used.

The construction of a wavelet packet basis is entirely defined by the wavelet scaling filter, hence its selection is critical. This filter solely determines the specific characteristics of the transform. In multicarrier systems, the primary characteristic of the waveform composing the multiplex signal is out-of-band energy. Though in an AWGN channel this level of out-of-band energy has no effect on the system performance thanks to the orthogonality condition, this is the most important source of interference when propagation through the channel causes the orthogonality of the transmitted signal to be lost. A waveform with higher frequency domain localization can be obtained with longer time support. On the other hand, it is interesting to use waveforms of short duration to ensure that the symbol duration is far shorter than the channel coherence time. Similarly, short waveforms require less memory, limit the modulation-demodulation delay and require less computation. Those two requirements, corresponding to good localization both in time and frequency domain, cannot be chosen independently. In fact, it has been shown that in the case of wavelets, the bandwidth-duration

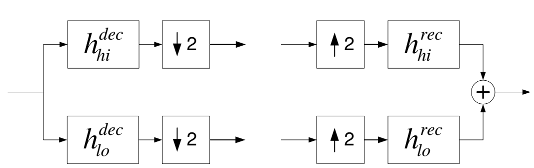

Fig. 2. Wavelet packet elementary block decomposition and reconstruction

Full name	Abbreviated name	Vanishing order	Length L_0
Haar	haar	1	2
Daubechie [17]	db$N$	N	$2N$
Symlets	sym$N$	N	$2N$
Coiflet	coif$N$	N	$6N$
Discrete Meyr	dmey	–	62

TABLE I

SUMMARY OF WAVELET FAMILY CHARACTERISTICS.

product is constant[1] [18]. This is usually referred to as the uncertainty principle.

We limit our performance analysis to WPT based on widely used wavelets such as those given in [17]. While there are numerous alternative wavelet families that could be used as well, a comparative study would deserve a separate publication by itself. As previously mentioned, we are essentially interested in wavelets leading to fast transforms through the tree algorithm. The primary wavelet family we have been using in our research is the one from Daubechie [17, p. 115], since it presents wavelets with the shortest duration. Furthermore, their localization in the frequency domain can be adequately adjusted by selecting their vanishing order, as it is illustrated by two of the curves plotted in Figure 4. The label *dbN* is used in the rest of this article to refer to WPT based on Daubechie wavelet of vanishing order N.

Further work carried out after her initial research led to a near symmetric extension of the *dbN* family, since it was a demanded feature in applications such as image compression [17, pp. 254-257]. Following the notation in [19], we will refer to this family as *Symlets*, denoted as *symN*. The last wavelet we will make reference to has been constructed at R. Coiffman demand for image processing applications [17, pp. 258-259] and will be denoted *coifN*. This wavelet is near symmetric, has $2N$ moments equal to 0 and has length $6N - 1$. Table I summarizes the characteristics of the wavelets and families mentioned above. Order 4 members of those two last families are as well plotted in Figure 4.

Finally, a minor difference between OFDM and WPM remains to be emphasized. In the former, the set of waveforms is by nature defined in the complex domain. WPM, on the other hand, is generally defined in the *real* domain but can be also defined in the *complex* domain, solely depending of the scaling and dilatation filter coefficients [20]. Since the most commonly encountered WPT are defined in the real domain, it has naturally led the authors to use pulse amplitude modulation for each subcarrier. It is nevertheless possible to translate the M real waveform directly in the complex domain. The resulting complex WPT is then composed of $2M$ waveforms forming an orthogonal set. While this is mathematically trivial, this simple fact has not been clearly emphasized in the telecommunication literature. This is of particular interest in systems where a baseband signal is desirable. The Asymmetrical Digital Subsciber Line (ADSL) standard falls for instance in this category [21]. Currently, a real baseband signal is constructed by using a $2M$-DFT fed with M conjugate pair modulation symbols [22]. An WPT of size M would therefore provide an equivalent modulation scheme at roughly half complexity.

A. Interesting features for wireless communication

In addition to the basic features already mentioned, the WPT presents interesting advantages from a system architecture point-of-view. For instance, the dependence of the WPT on a generating wavelet is a major asset in communication applications. An improved transmission integrity may be achieved with the aid of diversity [23, Section 14.4]. Space-, frequency-, and time-diversity are the most common physical diversities exploited. In addition, WPM provides a *signal* diversity which is similar to spread spectrum systems to some extent. Such diversity is in fact making joint use of time and frequency space. Practically, using two different generating wavelets allows us to produce two modulated signals that can transmitted on the same frequency band and suffer from reduced interference only. The amount of cross-correlation between the signals is directly dependent on the generatingwavelets chosen. The best method to be used to select suitable wavelets is a topic to be studied further. This particular feature could be exploited for instance in a cellular communication system, where different wavelets are used in adjacent cells in order to minimize inter-cell interference.

Another interesting feature of WPM is directly related to the iterative nature of the wavelet packet transform. The number of subcarriers in OFDM systems is usually fixed at design time, and it is especially difficult to implement an efficient FFT transform having a programmable size. This limitation does not exist in WPT since the transform size is determined by the number of iteration of the algorithm. From an implementation point of view, this can be made configurable without significantly increasing the overall complexity. This permits *on-the-fly* change of the transform size, which can be required for different reasons, for instance to reconfigure a transceiver according to a given communi-

[1] Both measurement in the frequency and time domain are taken as the domain in which most of the energy of the signal is localized.

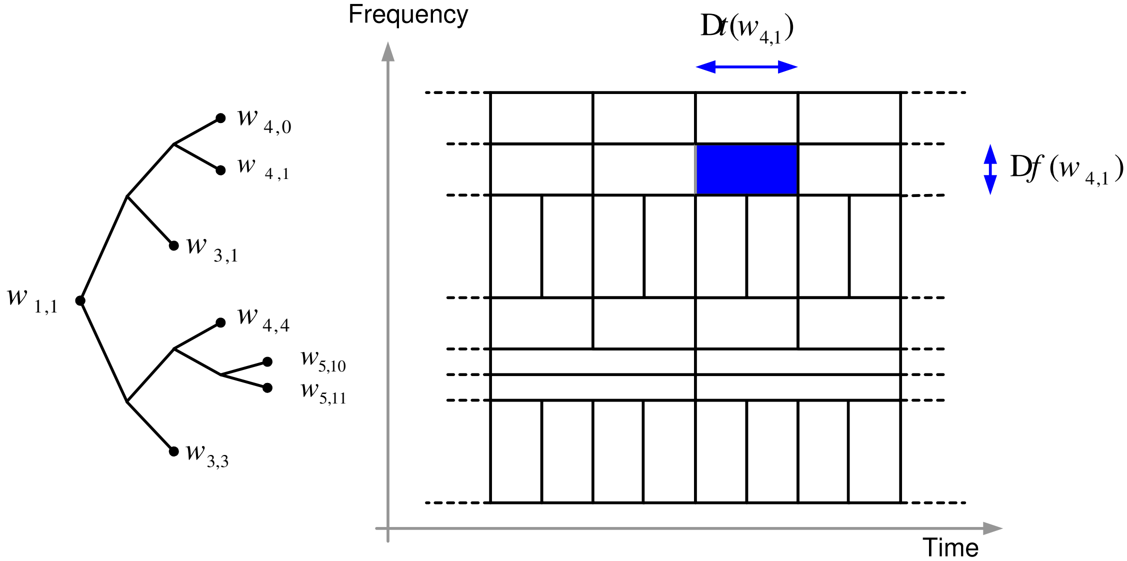

Fig. 3. Time-Frequency plane division with semi-arbitrary wavelet packet tree pruning

cation protocol[2] or, in the event of the appearence of cognitive radio [24], the transform size could be selected according to the channel impulse response characteristics, computational complexity and link quality.

The last property of the WPT is the semi-arbitrary division of the signal space. Wavelet packet transform still leads to a set of orthogonal functions, even if the construction iterations are not repeated for all sub-branches. From a multicarrier communication system perspective, this maps into having subcarriers of different bandwidth. Moreover, since each subcarrier has the same time-frequency (TF) plane area, the increase of bandwidth is bounded to a decrease of subcarrier symbol length. This is illustrated in Figure 3, where a non-regular decomposition tree is shown together with the corresponding time-frequency plane division. This characteristic of WPM has been referred to in the literature as a *multirate* system [25], although the term might be misleading in this case. While the transmission over the channel is effectively done at different symbol rates, the corresponding subcarrier throughput remains identical due to the constant subcarrier bandwidth-duration product. From a communication perspective, such a feature is usable for systems that must support multiple data streams with different transport delay requirements. A logical channel requiring lower transport delay could make use of a wider subcarrier, while some signalling information could be carried within a narrower bandwidth. Especially, those narrow subcarriers could be used for synchronization purpose in order to take advantage of longer symbols that require a small amount of bandwidth. Alternatively, the authors have carried out some research work on the selecting the transform tree according to the channel impulse response in the frequency domain. Preliminary results have not shown significant improvement in transform complexity versus link BER, but further work on this issue is still ongoing [26].

[2]OFDM-based standards make use of transform sizes ranging between 32 and 8192 subcarriers.

All together, WPM presents a much higher level of flexibility than current multicarrier modulation schemes. This makes WPM a candidate of choice for reconfigurable and adaptive systems such are the ones likely to compose the next generation of wireless communication devices.

III. Performance in various channel models

We analyze in this section the performance of WPM in diverse propagation channels. The results presented have been obtained by simulations only, due to the fact that no analytical expressions are available.

The derivation of a WPM link BER over an AWGN channel is a trivial issue since, identically to OFDM, orthogonality between subcarriers ensures the link performance is solely dependent on the constellation used on each subcarrier. The case of a Ricean channel can be derived similarly, assuming that the channel response is time-invariant for the duration of a whole WPM symbol. This is however a less conservative assumption than for OFDM since WPM symbols are several times longer than their OFDM counterparts. The case of transmission through the multipath channel hence remains to be studied more in depth.

A. WPM without equalization

Here we present on comparative results between WPM and OFDM schemes without channel equalization. While the absence of equalization in a real system would yield a poor performance in most channels, it is nevertheless of interest to consider this case here. The main motivation is to gain insight on the distortion caused by a given channel to the different modulated signals. Moreover, since optimal equalization schemes for both are different, the definition of equivalence between systems would require the choice of appropriate comparison criteria.

With the same concern of comparing equivalent systems, the OFDM reference system used in this section has no cyclic prefix. The addition of a prefix is indeed a technique aiming at rendering the multipath distortion easier to cancel and as such can be considered as a form of equalization [27, Section 18.6]. Since no similar artifact is available for WPM, the comparison is more fair if no cyclic prefix is used. In addition, the introduction of the cyclic prefix leads to a bandwidth efficiency loss and thus this would add to the difference between modulation schemes.

We choose to assume here the simple case of a time invariant 2-path channel model as introduced by Rummler [23, section 14-1-2]. The first path has unit power and the second path power is 3 dB lower. The relative delay of the second path τ is a simulation parameter. The BER versus the second path delay τ curves of WPM(db2), WPM(db6), and WPM(db10) schemes with 32 QPSK modulated subcarriers is shown in Figure 5. All curves are quite similar and thus the increase of the wavelet order has little impact on the performance. The main differences appear to be in the area of where τ is lower than 8 samples. For these values, the WPM scheme based on the higher order wavelet is less sensitive to actual delay.

The performance for two different OFDM schemes appears in Figure 5. For the reason discussed, the first has no cyclic prefix $(K = 0)$. The second one has a prefix of OFDM symbol duration normalized length $\frac{1}{4}$ which corresponds here to 8 samples. For τ lower or equal to 3 samples, the OFDM scheme without prefix performs slightly better than the WPM ones. For longer path delays, OFDM performs similarly to the WPM schemes, with a close resemblance to WPM(db2). OFDM with a cyclic prefix shows much better performance for path delays lower than the prefix duration since inter-symbol interference is removed[3]. When the path arrival delay τ exceeds the cyclic prefix length, the overall error rate increases dramatically, becoming even higher than with the other schemes at a path delay higher than 20.

Overall, the raw BER performance of WPM is identical to OFDM with the channel model assumed here. Additional results not reported here have shown equivalence for different frequency selective channels when no equalization is used. Hence the potential of improved performance compared to OFDM is bound to the design of an efficient equalization scheme for either modulation.

B. With equalization

The initial interest in multicarrier modulation schemes was the low complexity of the equalization they required. In multicarrier systems, equalization is usually divided into pre- and post-detection equalizers[4], according to their position respective to the core transform. In OFDM, a pre-equalizer is not required if the channel delay spread is shorter than the cyclic prefix. For longer delay spread, the pre-detection equalizer aims at shortening the apparent channel impulse response to a value lower or equal to the cyclic prefix duration. In the case it succeeds, the symbol at the input of the DFT is free from inter-symbol interference. A post-detection equalizer composed of a single tap per subcarrier is then sufficient to compensate for the channel distortion in the frequency domain. Though this low complexity structure is a major advantage of OFDM, it is limited by the fact that for a channel with long impulse response, the length of the prefix required leads to a significant loss in capacity and transmit power. For WPM however, the use of a cyclic prefix is impossible due to the overlapping of consecutive symbols. Hence both inter-symbol interference (ISI) and inter–symbols inter-carriers interference (ISCI) have to be cancelled by the equalization scheme. While no research work addresses this issue specifically for WPM, a comparison of the efficiency and complexity of pre-detection, post-detection, and combined equalization techniques for DWMT has been studied by Hawryluck *et al.* [29]. Research results show clearly that the best results are obtained by using both pre- and post-detection equalization. Due to the similarity with this modulation scheme, a similar approach is taken here and we separately analyze the performance of both types of equalization.

[3]The channel distortion is not cancelled since no equalization is assumed.

[4]As in [28], the terms pre- and post-detection equalization are prefered here to time- and frequency-domain equalizers, though they refer to the same functions

1) Pre-detection equalization: The objective of the pre-detection equalizer is slightly different between an overlapping multicarrier and an OFDM system making use of a cyclic prefix. While in the latter case, the equalizer attempts to shorten the perceived channel impulse response to a value lower than the cyclic duration, the equalization of WPM aims at minimizing the mean square error at its output.

We compare here the sensitivity of WPM and OFDM to imperfect equalization. To avoid the system's related issues concerning equalizer training, we will further assume that the equalizer taps have reached their near optimal values. Denoting the transfer function of the pre-detection equalizer $Q(z)$ and the equivalent discrete channel response $C(z)$, the overall link from modulator to demodulator has an equivalent transfer function of

$$H(z) = C(z)\,Q(z). \qquad (4)$$

Considering the case where the equalization method has converged to the value $\tilde{Q}(z)$, we choose to approximate $H(z)$ as

$$H(z) = 1 + \sum_{l=1}^{L_{eq}-1} q(l)\,z^{-l}. \qquad (5)$$

In the case of perfect equalization, all the coefficients $q(l)$ are null. To take into account imperfect equalization, we assume that the set $\{q(l)\}$ is normally distributed. We denoted σ_{eq}^2 the noise power in the L_{eq} equalizer taps indexed from 1 to $L_{eq}-1$. The sensitivity of WPM and OFDM can thus be compared as a function of the noise power σ_{eq}^2 in the equivalent transmission filter $H(z)$. Figure 6 displays the results of a WPM link for an average of 100 randomly generated channels as a function of the length L_{eq} of the resulting filter. The noise power σ_{eq}^2 has be set to -5 dBc in order to lead to a significant number of errors. All the schemes used in this simulation assume 16 subcarriers, 16QAM, and an AWGN channel with 20 dB SNR. Despite the symbols overlapping, WPM performs very similarly to OFDM in such scenarios. Moreover, no significant difference is perceived between the WPM schemes using different wavelet orders.

Alternatively, the effect of the noise power σ_{eq}^2 for the same modulation schemes for a fixed value of L_{eq} has been studied. While some differences exist for some given channel, the results confirm that all schemes asymptotically reach the same performance for any value of L_{eq} and σ_{eq}^2. Thus, WPM and OFDM exhibit identical sensitivity to pre-equalization errors. These results have been reached under the assumption that the equalizer taps have converged identically for both schemes. Further work is needed in order to verify the validity of this assumption for a particular equalizer training methods with a finite length training sequence.

2) Post-detection equalization: As underlined, post detection equalization is used in OFDM systems in the form of a single tap filter. These taps aim at inverting the channel transfer function in the frequency domain. For WPM, the structure of the post-detection equalizer is more complex since it has to remove both ISI and ISCI. Hence, a combined time-frequency equalization structure is required. Denoting η and ν as the number of equalizer taps in time and frequency domains respectively, we can express the estimated symbol $\tilde{\theta}_k^l$ at the output of the post-detection equalizer as

$$\tilde{\theta}_k^l = \sum_{\Delta k=-\frac{\nu}{2}+1}^{\frac{\nu}{2}-1} \sum_{\Delta l=-\frac{\eta}{2}+1}^{\frac{\eta}{2}-1} q(\Delta k, \Delta l)\,\hat{\theta}_{k+\Delta k}^{l+\Delta l}. \qquad (6)$$

The total number of operations per subcarrier symbol is $\eta\times\nu$, and the complexity grows rapidly if a complex structure is used. This is a major concern since with $L_0 - 1$ overlapping symbols, perfect equalization cannot be reached for $\eta \leq (L_0 - 1)$. Such equalization complexity is impractical in a real system, hence we limit our analysis to smaller equalizer sizes that give a BER sufficiently low to permit an outer error correction code to reach a good performance.

We choose to compare the performance of the equalization schemes for a typical channel with an exponentially decaying profile. The decay factor τ_0 is taken equal to 1 and the total number of paths is limited to 4. For this channel, the equalizer taps have been calculated through the steepest descent LMS algorithm [23, Section 11-1-2]. The iteration step Δ has been taken equal to 10^{-3} and the number of training symbols has be chosen to ensure the algorithm has converged.

The performance of the equalizer as a function of its number of taps η in time domain is given in Figure 7. The modulation schemes selected here are WPM(coif1) and WPM(coif4), in order to allow comparison of the effect of better frequency domain localization. Considering first the case where the equalizer does not take into account the symbols on adjacent subcarriers (i.e. $\nu = 1$), the increase of η brings only very limited improvement, though the difference is slightly more significant for the WPM(coif4) scheme. When the two adjacent subcarrier symbols are used in the equalizer, increasing η from 1 to 3 taps leads to a larger interference reduction of about of 1 dB. A further increase of both ν and η bring no improvement in the WPM(coif1) scheme, and only limited interference reduction in the case of the more frequency localized WPM(coif4) scheme. Furthermore, it is interesting to notice that, while all the modulation schemes have shown identical performance when no equalization is used, the addition of a single tap equalizer per subcarrier reveals the extra robustness of the WPM(coif4) in comparison with the WPM(coif1) scheme.

Overall, the post-equalization of a WPM signal requires at least a 3×3-tap structure to remove the largest part of the interference caused by multipath propagation. Less frequency localized WPM schemes require an even more complex structure due to an increased inter-subcarrier interference level. This of course leads to a complexity penalty when compared with OFDM schemes where a cyclic prefix can be used and only one tap per subcarrier is needed. Hence, the complexity gain of WPM emphasized later in this article is reduced when equalization is considered.

3) Discussion on equalization: The results obtained emphasise that equalization of WPM in multipath channels is more complex than that of OFDM. This is essentially due to the use of a cyclic prefix that gives an edge to OFDM, when compared to overlapping multicarrier schemes such as WPM. Thus, the raw link performance of WPM and OFDM is equivalent only when the cyclic prefix is not a practical solution. This corresponds to the case where the channel impulse response is too long to allow reasonable performance with a low number of subcarriers. In the latter situation, the system complexity of the WPM and OFDM schemes is roughly equivalent since OFDM requires more than a single tap post-equalizer to remove multipath interference.

IV. Robustness to interferers, non-linear distortion and sampling offsets

In addition to the performance of a modulation scheme in the given propagation condition, its sensitivity to implementation impairments is a characteristic that makes it a candidate for an actual communication system. Hence we report in this section the performance of WPM in the presence of some of the implementation impairments that are critical in OFDM systems: the sensitivity to non-linear distortion and sampling instant error.

A. Non-linear signal distortion effects

OFDM modulated signals are known to exhibit a high peak to average power ratio (PAPR) [30], [31]. This implies that power amplifiers must have a high dynamic range in order to avoid causing distortion. This is of concern in power-limited communication devices, since the energy consumption and cost of radio-frequency amplifiers increases with the need for higher linear dynamic range. No work on this issue has been reported in the literature specifically for WPM, though the use of a wavelet packet derived pulse over non-linear satellite communication channel has been addresses by Dovis *et al.* [32]. Thus, we evaluate the actual sensitivity of different WPM schemes to non-linear distortion.

A model of the non-linear amplifier has to be chosen. Different approaches have been used in the literature [33], [34]. For most complex models, the numerous parameters have to be derived from measurement on an actual device. To keep the analysis here more generic, we will assume a simple non-linear, memoryless power amplifier model. For most applications using medium and lower transmit power, phase distortion can be assumed to be null. Hence, only the amplitude distortion is assumed to be significant. Such a simple model is characterized by a transfer function of the form [30, section 6.3.1]:

$$y\big(x(t)\big) = x(t)\left[1 + \left(\frac{x(t)}{\gamma}\right)^{2p}\right]^{-\frac{1}{2p}} \tag{7}$$

with $y(t)$ the output signal, $x(t)$ the input signal, and γ and p are the backoff margin and the linearity order of the model respectively. A higher value of p leads to an output versus input amplitude response, where the output amplitude is smoothly reaching its saturation value as the input power increases. For an average quality radio-frequency power amplifier, the best fit value for p is found to be between 2 and 3 [30, section 6.3.1]. Note that γ is referred to as the backoff margin and is defined as the difference between the average and peak power of the amplifier[5].

Simulation results are displayed in Figure 8, where the link BER for WPM(db2), WPM(sym2), WPM(coif2), WPM(dmey), and OFDM schemes is plotted as a function of the backoff margin γ. An AWGN channel with 20 dB SNR has been assumed. WPM schemes require a backoff value 1.5 dB higher than OFDM to achieve equal performance at 10^{-4} BER. Among the WPM schemes, WPM(dmey) performs best, requiring about 0.4 dB less power than WPM(db2) and WPM(sym2) schemes at 10^{-4} BER. In order to reach insignificant BER degradation due to non-linearity, the OFDM signal requires a 5 dB backoff value. WPM schemes need a slightly higher value of 6 dB. This 1 dB difference is rather low and

[5]This differs from the definition commonly used where the backoff margin is the difference between the amplifier average power operating point and -1 dB compression point.

thus should not lead to any significant complexity variation in the radio-frequency section of a modem using either modulation scheme.

B. Sampling phase offset

Multicarrier modulation signals are by nature much more sensitive to synchronization errors than single carrier ones [35]. We focus here on the effect of a non-ideal sampling instant.

The interference caused by a sampling phase error $\Delta\tau$ is of three kinds. There is a gain loss in the recovery of the symbol of interest, an inter-carrier interference term, and an inter-symbol inter-carrier interference contribution. This last term originates from the symbol overlapping and thus does not exist in OFDM. Hence, the sensitivity of WPM is expected to be higher than for OFDM. In addition, the BER degradation depends on the auto-correlation of each subcarrier waveform, which differs between subcarriers. Overall, the BER of the multicarrier signal can be obtained as the average of the BER over each individual subcarrier. Since no analytical closed form solution is available, the sensitivity of each modulation scheme to sampling instant error has been obtained by simulation. Figure 9 reports link BER as a function of the sampling phase offset normalized to the sampling period. For this particular simulation, the channel is modelled as AWGN with 20 dB SNR. As it was expected due to the overlapping of symbols, WPM is more sensitive than OFDM to an imperfect sampling instant. A BER of 10^{-4} is achieved for OFDM at a normalized sampling error of 27%, while WPM(db2) requires less than 21%. For the two other WPM schemes, the error tolerated is slightly lower, with a maximum of about 18%.

Additional results on the sensitivity of WPM to sampling instant error as a function of the wavelet order is given in Figure 10. The differences between the different order of the *coif* wavelet are limited, except for the lower order one which tolerates a 2% higher phase error than the others. There is no direct relation between the order and sensitivity level. Simulation results for *sym* wavelets which are not shown here have led to identical conclusions.

Overall, the higher sensitivity of WPM implies that a more robust sampling instant synchronization scheme is required in comparison with OFDM. On the other hand, the multi-resolution structure of the wavelet packet waveforms offer the opportunity for improved synchronization algorithms [13], [12]. This has been exploited for instance in the case of WPM by Luise *et al.* [14], where the proposed synchronization scheme has shown enhanced performance that could compensate for the sensitivity of WPM to synchronization errors. No implementation of the algorithm has been reported and therefore its performance in the presence of imperfect estimation remains to be studied.

C. In presence of a narrow band interferer

With today's growing use of wireless systems, the radio-frequency spectrum is becoming more and more congested by communication signals. In such a condition, future modulation schemes have to cope not only with channel distortion, but also with interference originating from other sources as well. Moreover, multicarrier modulation is very likely to encounter in-band interfering signal since it is usually best suited for wideband communication systems.

We assume the case of a WPM link communicationg over a AWGN channel and exposed to an in-band, un-modulated signal superimposed on the signal of interest at the receiver. We denote P_{dist} as the power of the disturbing signal and f_{dist} as its frequency. The amount of interference endowed by each subcarrier k can thus be approximated as [28]

$$P_I = P_{dist} \sum_{k=0}^{M-1} \left|\Phi_k(f_{dist})\right|^2 \qquad (8)$$

where Φ_k in the Fourier transform of φ_k. In the case of waveforms with null out-of-band energy, the disturbance will be limited to the subcarrier whose band includes the frequency f_{dist}. With an actual system, additional disturbance is caused by the side lobes of the adjacent subcarriers. The side lobe energy level decreasing with the order of the wavelet, the sensitivity of WPM to a single tone disturber can thus be reduced by increasing the order of the generating wavelet.

Results obtained through simulation are shown in Figure 11, where the BER performance of WPM(db2), WPM(db8), and OFDM links are plotted as a function of the disturber normalized frequency F_{dist}. A 16-subcarrier scheme with 16QAM modulated symbols has been chosen to point out the effect of the disturber. The disturber has power equal to the signal of interest, i.e. $P_{dist} = 0\ dBc$. The curves obtained for all modulation schemes show clearly that the level of interferance is highly dependent on the actual disturber frequency. Hence, the curve for OFDM shows clearly the higher BER when the disturber frequency corresponds to the center frequency of one subcarrier. The WPM schemes show a similar effect but with a smoother curve. Considering the average over the whole frequency band, WPM(coif5) and WPM(dmey) outperform OFDM. The WPM(coif1) scheme however shows more degradation than OFDM. Overall, the

WPM schemes seem to be able to perform better than OFDM when a wavelet with sufficiently low out-of-band energy is used. For a given wavelet, there appear to be a gain in robustness with the increase in generating wavelet order.

Additional results are presented in Figure 12 where the link BER of the same schemes as previously are given as a function of the disturber relative power P_{dist}. Its normalized frequency has been arbitrary chosen to be 0.1666, which can be verified from the previous figure to corresponds to the case where WPM leads to an average gain in comparison with OFDM. The WPM(coif1) scheme performs overall quite similarly to OFDM, but with a robustness to disturber about 2 dB lower. The WPM(coif5) and WPM(dmey) schemes, on the other hand, provide a much higher robustness than OFDM for a disturber power of up to -15 dB. Past this threshold, it is noticeable that OFDM has instead a lower sensitivity, being undisturbed for P_{dist} lower than -20dB while the two WPM schemes require about 2 dB less.

In general, the degradation in terms of BER of the WPM signal due to a single tone disturber is highly dependent on both its frequency and power. The results obtained have nevertheless shown that WPM schemes are capable of high immunity to disturbance when higher order wavelets are selected.

V. Implementation complexity estimates

The structure of a WPM-based transceiver is very similar to that of OFDM. This section reports on the implementation complexity of the WPT since it is the building block in which the systems differ the most[6]. Three alternative architectures are considered that differ in the implementation of the elementary blocks.

We first consider the direct implementation of the elementary blocks as they are shown in Figure 2. In the case of the forward transform, the elementary blocks consist simply of two up-samplers and two FIR filters of length L_0. For the inverse transform, an additional adder in required to combine the output of the two filters, so the overall number of operations is slightly lower. We consider further the forward transform only, since the case of the reverse one can be easily derived from it. With filters of length L_0, the number of operations required by each filter is actually equivalent to the complexity required by a $L_0/2$-tap long filter thanks to the zero values samples inserted by the upsampler. Hence, the number of operations per input sample required by one elementary block is:

$$\mathcal{P}_{WPT}^{direct} = \begin{cases} 4\left\lceil \frac{L_0}{2} \right\rceil - 2 & adds \\ 4\left\lceil \frac{L_0}{2} \right\rceil & mults \end{cases} . \quad (9)$$

where $\lceil \cdot \rceil$ denotes the smallest integer higher than its argument.

Overall, a transform of size $M = 2^j$ is composed of $\mathcal{N}(j)$ elementary blocks per stage j. Denoting $\mathcal{R}(j)$ as the input rate of the block of stage j, the number of operations for the stage j:

$$\mathcal{C}_{WPT}(j) = \mathcal{N}(j)\,\mathcal{R}(j)\,\mathcal{P}_{WPT}^{direct} \quad (10)$$
$$\text{with} \begin{cases} \mathcal{N}(j) &=& 2^{j-1} \\ \mathcal{R}(j) &=& 2^{J-j} \end{cases} ,$$

where the transform input signal rate $\mathcal{R}(J)$ is assumed to be equal to unity. The overall number of operations for the WPT is then

$$\mathcal{C}_{WPT}(J) = \sum_{j=1}^{J} 2^{J-1}\,\mathcal{P}_{WPT}^{direct} \quad (11)$$
$$\mathcal{C}_{WPT}(J) = \begin{cases} (2^J-1)\left(4\left\lceil \frac{L_0}{2} \right\rceil - 2\right) & adds \\ (2^J-1)\left(4\left\lceil \frac{L_0}{2} \right\rceil\right) & mults \end{cases} .$$

This computational complexity can be further compared to that required by the DFT used in OFDM systems. We assume here the implementation of the DFT considered in [36], leading to a complexity of

$$\mathcal{C}_{DFT}(J) = \begin{cases} 2^J\,(2^J-1) & adds \\ J\,2^J & mults \end{cases} . \quad (12)$$

Figure 13 displays the complexity of the WPT relative to the DFT in terms of additions and multiplications. The number of additions is lower for the WPT for transform sizes higher than 8 with $L_0 = 4$, and for a transform size greater then 32 in the case where a longer filter with $L_0 = 16$ is used. The number of multiplications on the other hand is generally higher for the WPT. With the shorter filter ($L_0 = 4$), the number of multiplications is equal for both 256-point transforms. In the case of an implementation with a generic processor, the cost of both operations is identical. Hence, Figure 14 shows the total number of operations required by the WPT, again relative to the DFT for different lengths L_0 of the wavelet filter. The WPT complexity is higher for small size transforms, but decreases as the size of the transform increases. Hence, for a transform size over 64, the number of operations for the WFT is lower than for the DFT, even in the case of $L_0 = 16$.

Overall, the WPT implementation complexity is on the same order of the one required by the

[6]Optimal systems might differ as well in equalization and synchronization schemes.

DFT, and might even be lower for medium size transforms and wavelet filters of moderate length. The repartition between the addition/multiplication operations leads to the conclusion that the complexity is in favor of the WPT in the case of an implementation in a general purpose signal processor. In the case of a hardware implementation, the higher complexity of a multiplier in comparison with an adder is likely to reduce the difference between the two schemes. The iterative structure of the WPT is nevertheless very well suited to hardware implementation [37]. For each stage of the transform, the product of the number of elementary block by their processing rate is constant and equal to 2^{J-1}. A complete J stage transform can be performed by a single occurrence of the elementary block running at a clock speed $J\,2^{J-1}$ times the symbol rate or by 2^{J-1} instantiations running at J times the symbol rate. This provides an appreciable flexibility in trading-off speed versus silicon area, a feature that is suitable for the future generation of reconfigurable systems.

VI. Conclusions and further research topics

We reviewed in this article the advantages of using WPM for multicarrier communication systems. An in-depth comparison between this new scheme and OFDM has been reported. The performance of WPM has been shown to be identical to the latter in several reference wireless channels, though at a higher cost of equalization in multipath channels. Additional results taking into consideration effects of some non-ideal elements of the system have shown that WPM is slightly more sensitive than OFDM to these commonly encountered types of distortion. A comparison of the core transforms has shown the low complexity potential of WPM based systems.

Overall, the performance results of WPM lead us to conclude that this new modulation scheme is a viable alternative to OFDM to be considered for today's communication systems. OFDM remains nevertheless a strong competitor thanks to its capability to cope with multipath effects efficiently. With the demand for ever higher bandwidth, the use of cyclic prefix will likely become restricted, and hence OFDM and WPM will present equalization complexity of the same order.

The major interest of WPM nevertheless resides in its ability to fulfill the wide range of requirements of tomorrow's ubiquitous wireless communications. Hence, WPM has the strong advantage of being a generic modulation scheme whose actual characteristics can be widely customized to fulfill the various requirements of advanced mobile communications. This generic modulation has the potential of becoming a unique multicarrier communication scheme used by devices with various constraints. A single scheme could thus be used to communicate with small, resource-limited devices as well as high capacity, multimedia capable nodes. While considering such a scenario, it appears that the full capability of WPM can only be exploited by a system having the capacity to determine the best suited configuration. Hence, this fully supports the fact that WPM is the modulation of choice for smart, environment and resource aware wireless systems.

Again, the authors believe that the most interesting feature of WPM is its inherent flexibility. While such flexibility cannot be fully exploited with current systems and technologies, it is foreseen that future generation systems will be able to provide the degree of intelligence required to achieve optimal performance in a given environment, at a given quality, and for a limited energy consumption. WPM is then likely to become a strong competitor to multicarrier systems currently in use.

Some issues still remain in the way of the widespread use of WPM. Although its principle has been proposed for more than a decade, WPM has not been studied in depth yet. Its similarity to both OFDM and overlapping multicarrier systems such as CMFB allows the use of the work done for the latter as a basis for the research developments. This has been the approach taken in this paper in order to evaluate the potential of this new modulation scheme. The first area where a large amount of work remains to be done is equalization. The overlapping of symbols causes a significant amount of interference that requires an dedicated equalization scheme to be studied. A study on synchronization of the WPM signals in both time and frequency domains would most probably lead to efficient algorithms, due to the multi-resolution nature of the multiplexed signal.

Finally, it is important to underline that wavelet theory is still developing. Since the use of wavelet packets in telecommunications has been mainly studied by communications engineers, an important potential for improvements is possible if some of the specific issues are addressed from a mathematical point-of-view. Wavelet based communication systems have already shown a number of advantages over conventional systems. It is expected that more is still to be pointed out as the knowledge of this recently proposed scheme gains more interest within both the wireless communication industry and research community.

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REFERENCES

[1] J. A. C. Bingham, "Multicarrier modulation for data transmission: an idea whose time has come," *IEEE Communications Magazine*, vol. 28, no. 5, pp. 5–14, May 1990.

[2] A. Akansu, P. Duhamel, X. Lin, and M. de Courville, "Orthogonal transmultiplexers in communications: a review," *IEEE Transactions on Signal Processing*, vol. 46, no. 4, pp. 979–995, April 1998.

[3] R. Lackey and W. Upma, "Speakeasy: the military software radio," *IEEE Communications Magazine*, vol. 33, no. 5, pp. 56–61, May 1995.

[4] B. G. Negash and H. Nikookar, "Wavelet-based multicarrier transmission over multipath wireless channels," *Electronics Letters*, vol. 36, no. 21, pp. 1787–1788, October 2000.

[5] C. J. Mtika and R. Nunna, "A wavelet-based multicarrier modulation scheme," in *Proceedings of the 40^{th} Midwest Symposium on Circuits and Systems*, vol. 2, August 1997, pp. 869–872.

[6] G. W. Wornell, "Emerging applications of multirate signal processing and wavelets in digital communications," in *Proceedings of the IEEE*, vol. 84, 1996, pp. 586–603.

[7] N. Erdol, F. Bao, and Z. Chen, "Wavelet modulation: a prototype for digital communication systems," in *IEEE Southcon Conference*, 1995, pp. 168–171.

[8] A. R. Lindsey and J. C. Dill, "Wavelet packet modulation: a generalized method for orthogonally multiplexed communications," in *IEEE 27^{th} Southeastern Symposium on System Theory*, 1995, pp. 392–396.

[9] A. R. Lindsey, "Wavelet packet modulation for orthogonally multiplexed communication," *IEEE Transaction on Signal Processing*, vol. 45, no. 5, pp. 1336–1339, May 1997.

[10] N. Suzuki, M. Fujimoto, T. Shibata, N. Itoh, and K. Nishikawa, "Maximum likelihood decoding for wavelet packet modulation," in *IEEE Vehicular Technology Conference (VTC)*, 1999, pp. 2895–2898.

[11] S. Gracias and V. U. Reddy, "An equalization algorithm for wavelet packet based modulation schemes," *IEEE Transactions on Signal Processing*, vol. 46, no. 11, pp. 3082–3087, November 1998.

[12] F. Daneshgaran and M. Mondin, "Clock synchronisation without self-noise using wavelets," *Electronics Letters*, vol. 31, no. 10, pp. 775–776, May 1995.

[13] ——, "Symbol synchronisation using wavelets," in *IEEE Military Communications Conference*, vol. 2, 1995, pp. 891–895.

[14] M. Luise, M. Marselli, and R. Reggiannini, "Clock synchronisation for wavelet-based multirate transmissions," *IEEE Transactions on Communications*, vol. 48, no. 6, pp. 1047–1054, June 2000.

[15] G. Strang and T. Q. Nguyen, *Wavelet and filter banks*. Wellesley-Cambridge Press, 1996.

[16] S. Mallat, *A wavelet tour of signal processing*, 2nd ed. Academic Press, 1999.

[17] I. Daubechies, *Ten lectures on wavelets*. SIAM, CBMS Series, April 1992.

[18] C. K. Chui, *An introduction to wavelets*. Academic Press, 1992, vol. 1.

[19] The MathWorks Inc., "Wavelet toolbox user's guide, version 2," September 2000.

[20] X. Q. Gao, T. Q. Nguyen, and G. Strang, "On two-channel orthogonal and symmetric complex-valued FIR filter banks and their corresponding wavelets," in *Proceedings of IEEE International Conference on Acoustics, Speech and Signal Processing*, 2002, pp. 1237–240.

[21] W. Y. Chen, *DSL: Simulation techniques and standards development for digital subscriber line systems*. Macmillan Technical Publishing, 1998.

[22] ANSI Committee, "ANSI T1.413-1995: Network and Customer Installation Interfaces - Asymmetric Digital Subscriber Line (ADSL) Metallic Interface," ANSI Standard Committee T1E1.4, 1995.

[23] J. G. Proakis, *Digital communications*, 3rd ed. Mc Graw Hill international editions, 1995.

[24] J. Mitola and G. Q. Maguire, "Cognitive radio: Making software radios more personal," *IEEE Personal Communications*, vol. 6, no. 4, pp. 13–18, August 1999.

[25] W. Yang, G. Bi, and T.-S. P. Yum, "Multirate wireless transmission system using wavelet packet modulation," in *Proceedings of Vehicular Technology Conference (VTC)*, vol. 1, 1997, pp. 368–372.

[26] D. Daly, "Efficient multi-carrier comunication on the digital subscriber loop," Ph.D. dissertation, Dept. of Electrical Engineering, University College Dublin, May 2003.

[27] L. Hanzo, W. Webb, and T. Keller, *Single- and multicarrier quadrature amplitude modulation - Principles and applications for personnal communications, WLANs and broadcasting*. John Wiley & Sons, 2000.

[28] S. D. Sandberg and M. A. Tzannes, "Overlapped discrete multitone modulation for high speed copper wire communications," *IEEE Journal on Selected Area in Communications*, vol. 13, no. 9, pp. 1571–1585, December 1995.

[29] M. Hawryluck, A. Yongacoglu, and M. Kavehrad, "Efficient equalization of discrete wavelet multi-tone over twisted pair," in *Proceedings of Broadband Communications, Accessing, Transmission and Networking Conference*, 1998, pp. 185–191.

[30] R. van Nee and R. Prasad, *OFDM for wireless multimedia communications*. Artech house, 2000.

[31] A. R. S. Bahai and B. R. Saltzberg, *Multicarrier digital communications*. Kluwer Academic / Plenum Publishers, 1999.

[32] F. Dovis, M. Mondin, and F. Daneshgaran, "Performance of wavelet waveforms over linear and non-linear channels," in *Proceedings of Wireless Communications and Networking Conference (WCNC)*, 1999, pp. 1148–1152.

[33] M. Tummla, M. T. Donovan, B. E. Watkins, and R. North, "Voltera series based modeling and compensation of nonlinearities in high power amplifers," in *Proceedings of International Conference on Acoustics, Speech and Signal Processing*, vol. 3, 1997, pp. 2417–2420.

[34] G. Chrisikos, C. J. Clark, A. A. Moulthrop, M. S. Muha, and C. P. Silva, "A nonlinear ARMA model for simulating power amplifers," in *IEEE MTT-S International Microwave Symposium Digest*, vol. 2, 1998, pp. 733–736.

[35] M. Speth, S. Fechtel, G. Fock, and H. Meyr, "Broadband transmission using OFDM: system performance and receiver complexity," in *International Zurich Seminar on Broadband Communications*, February 1998, pp. 99–104.

[36] G. Bachman, L. Narici, and E. Beckenstein, *Fourier and wavelet analysis*. Springer, 2000.

[37] A. Jamin and P. Mähönen, "FPGA implementation of the wavelet packet transform for high speed communications," in *International Conference on Field Programmable Logic (FPL)*, Montpellier, France, September 2002.

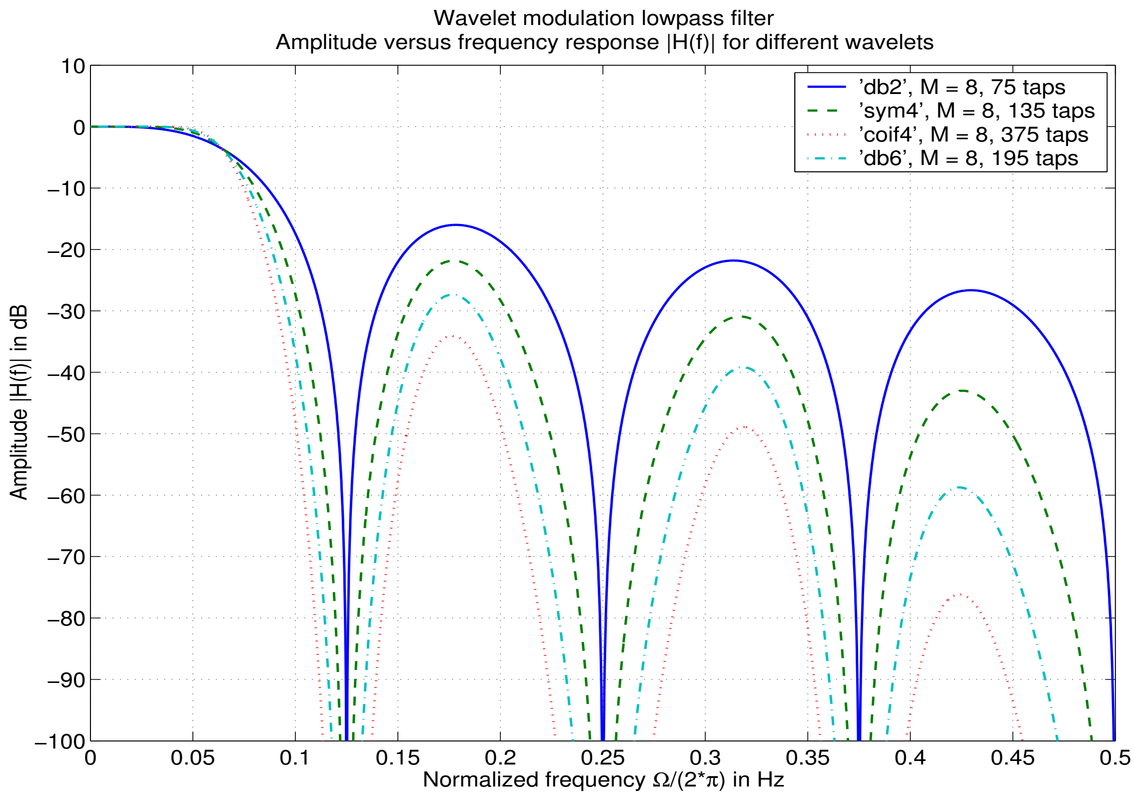

Fig. 4. Frequency domain localization of different wavelets

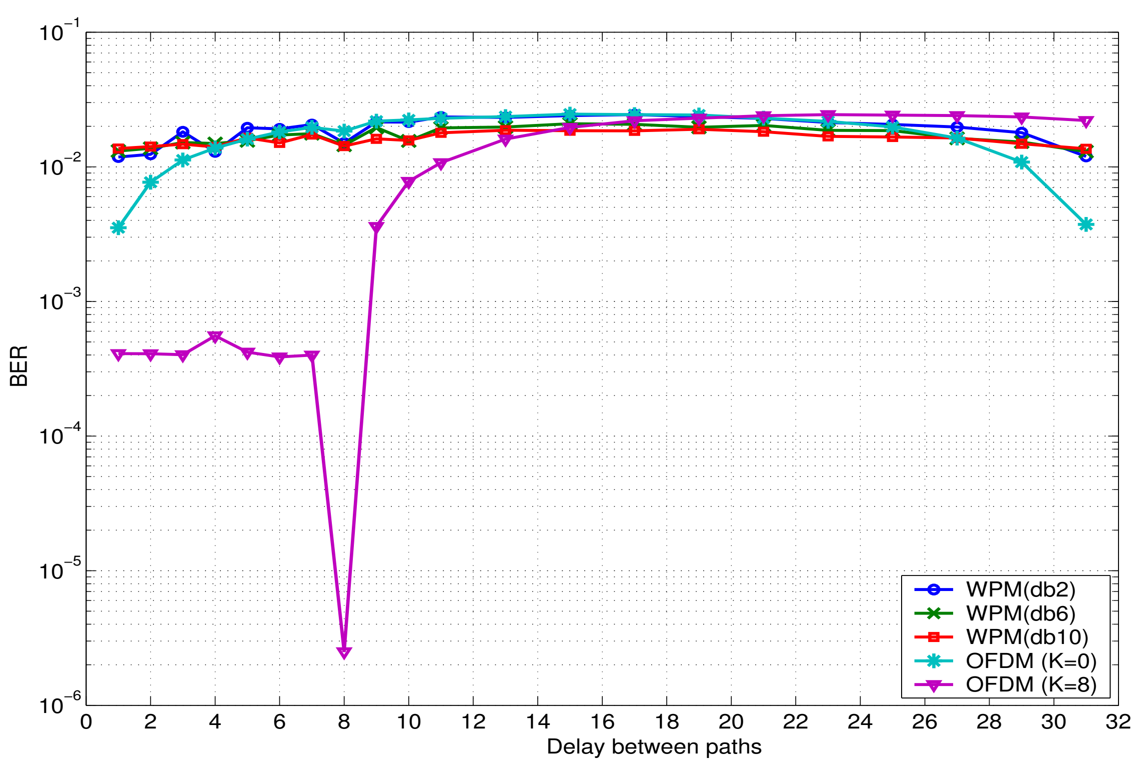

Fig. 5. Performance of WPM versus OFDM in a 2-path time-invariant channel. BER is plotted as a function of the delay of arrival of the second path. The delayed path relative power of −3 dBc and the SNR is 20 dB

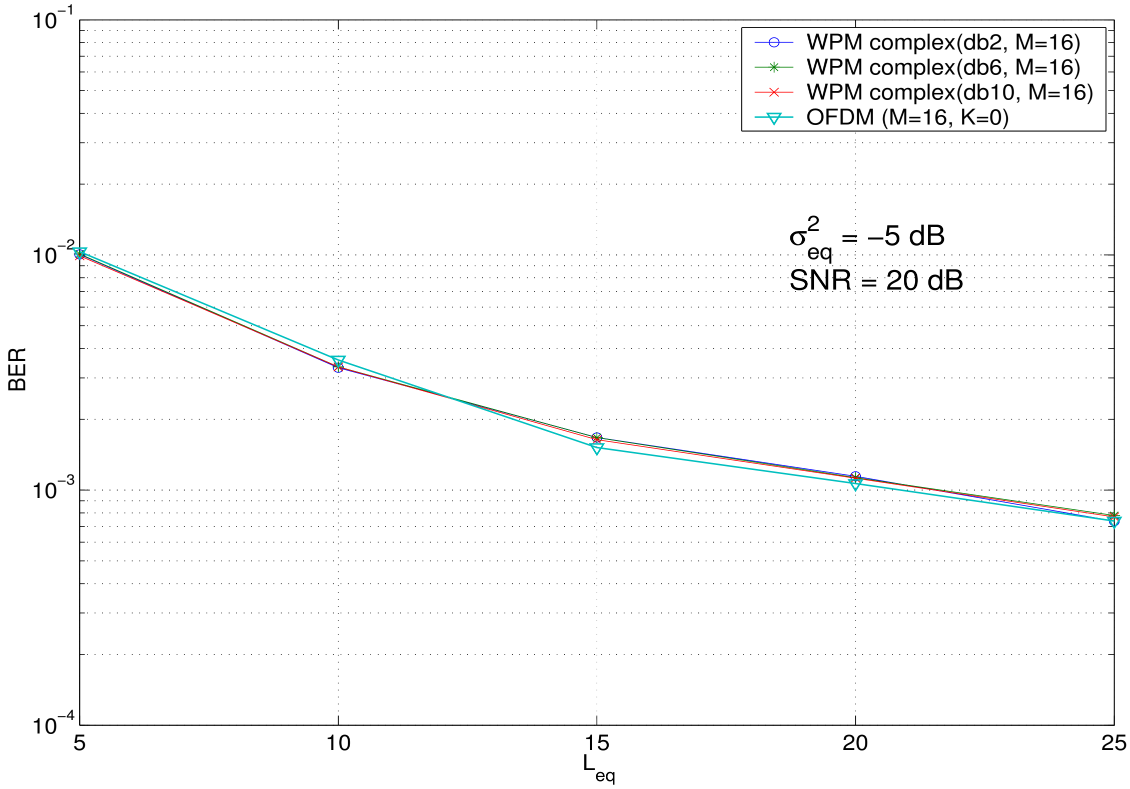

Fig. 6. BER performance of WPM and OFDM schemes with noisy equivalent impulse response, as a function of the length L_{eq} of imperfectly equalized channel response, at $\sigma_{eq}^2 = -5dBc$.

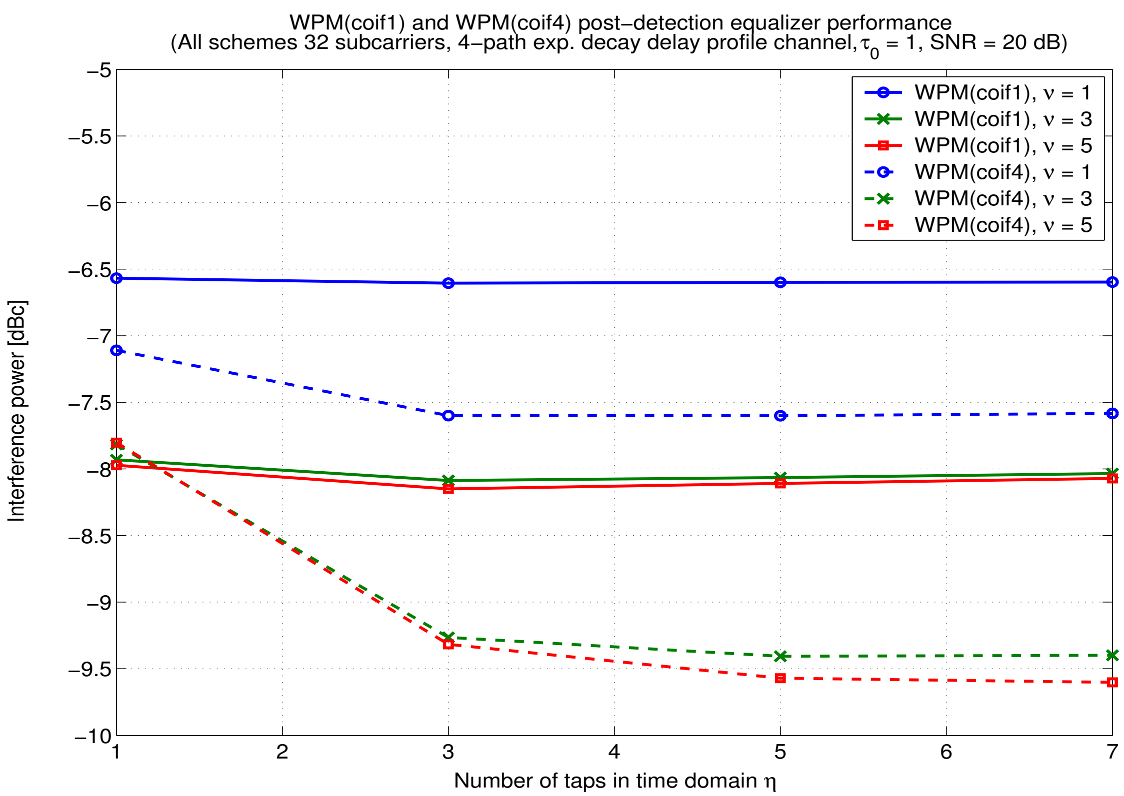

Fig. 7. Comparative performance of WPM(coif1) and WPM(coif4) schemes with post-detection equalization, as a function of the number of equalizer taps η in the time domain and with the number of taps ν in the frequency domain as a parameter.

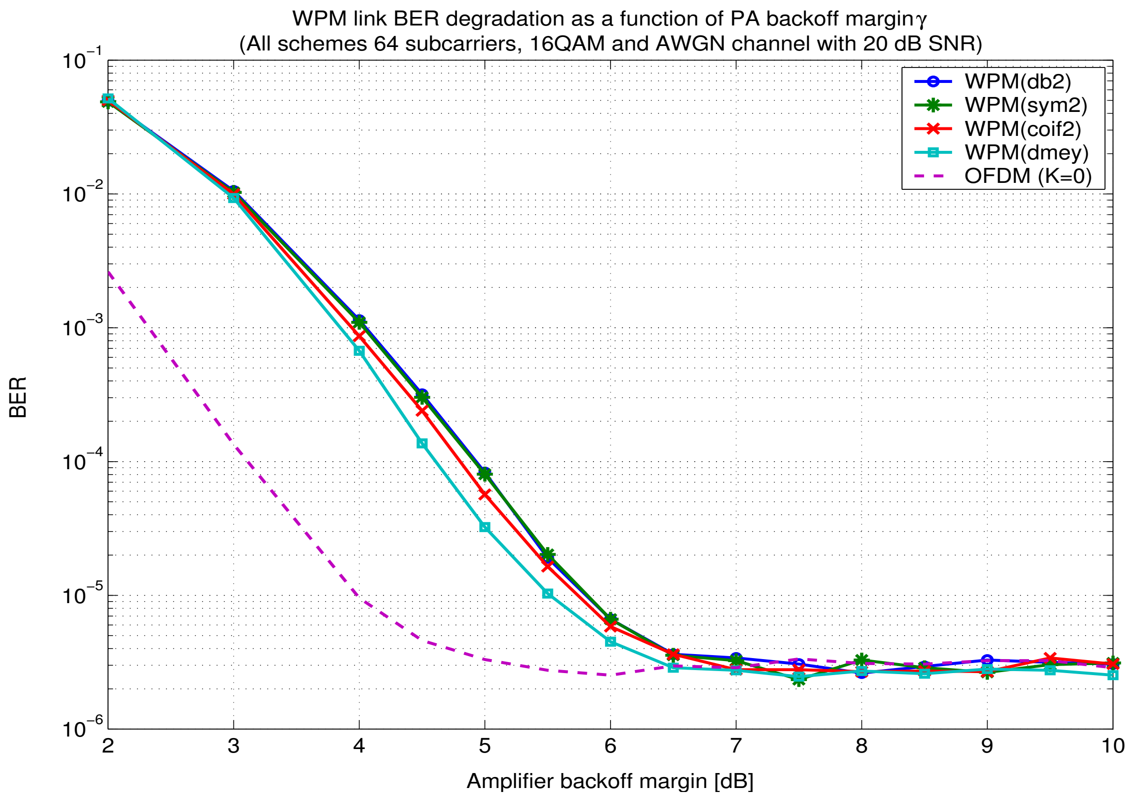

Fig. 8. BER versus power amplifier backoff margin γ for different WPM schemes and OFDM.

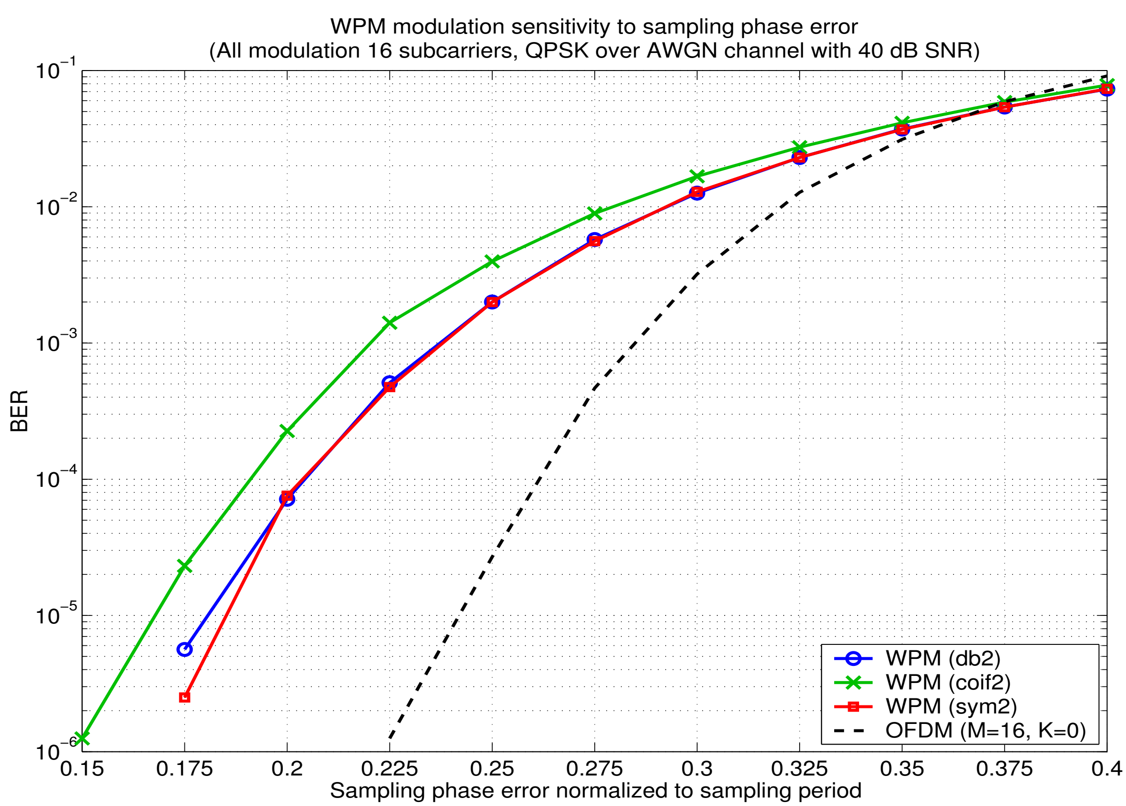

Fig. 9. Sensitivity of different WPM schemes versus OFDM schemes to sampling phase error, expressed as the link BER versus the normalized sampling phase error.

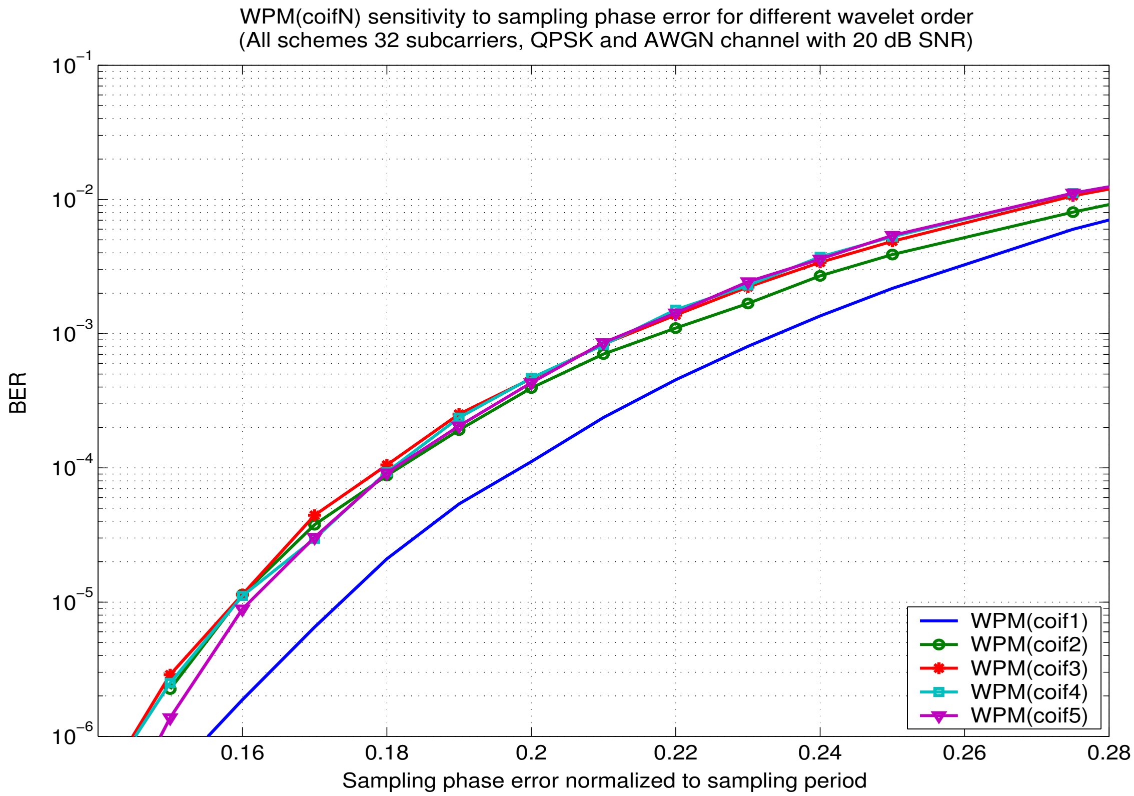

Fig. 10. Sensitivity of WPM(coif$N$) schemes to sampling phase error as a function of the wavelet order, expressed as the link BER versus the normalized sampling phase error.

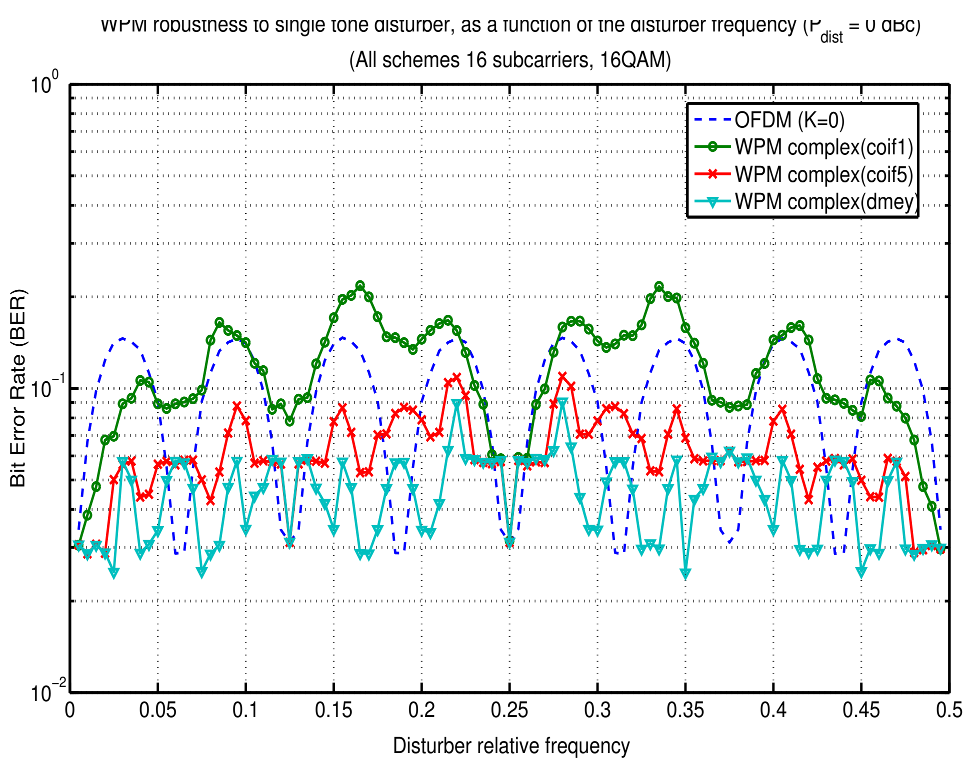

Fig. 11. Link BER in the presence of a single tone disturber as a function of the disturber frequency, for WPM(coif1), WPM(coif5), WPM(dmey), and OFDM schemes.

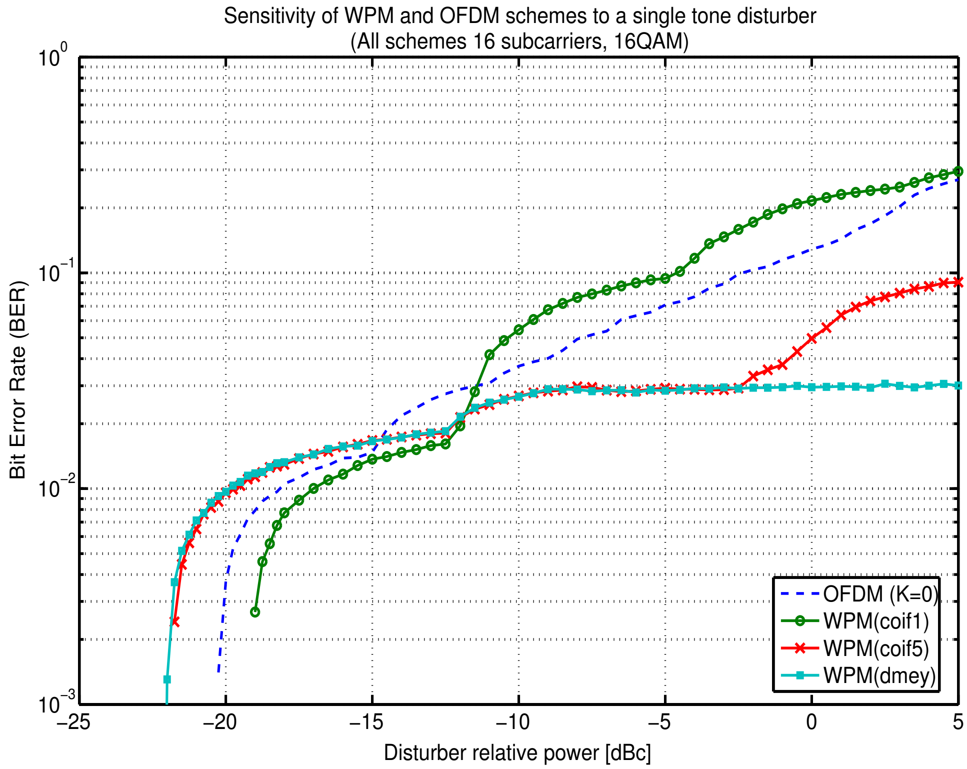

Fig. 12. Link BER in the presence of a single tone disturber as a function of the disturber power, for WPM(coif1), WPM(coif5), WPM(dmey), and OFDM schemes.

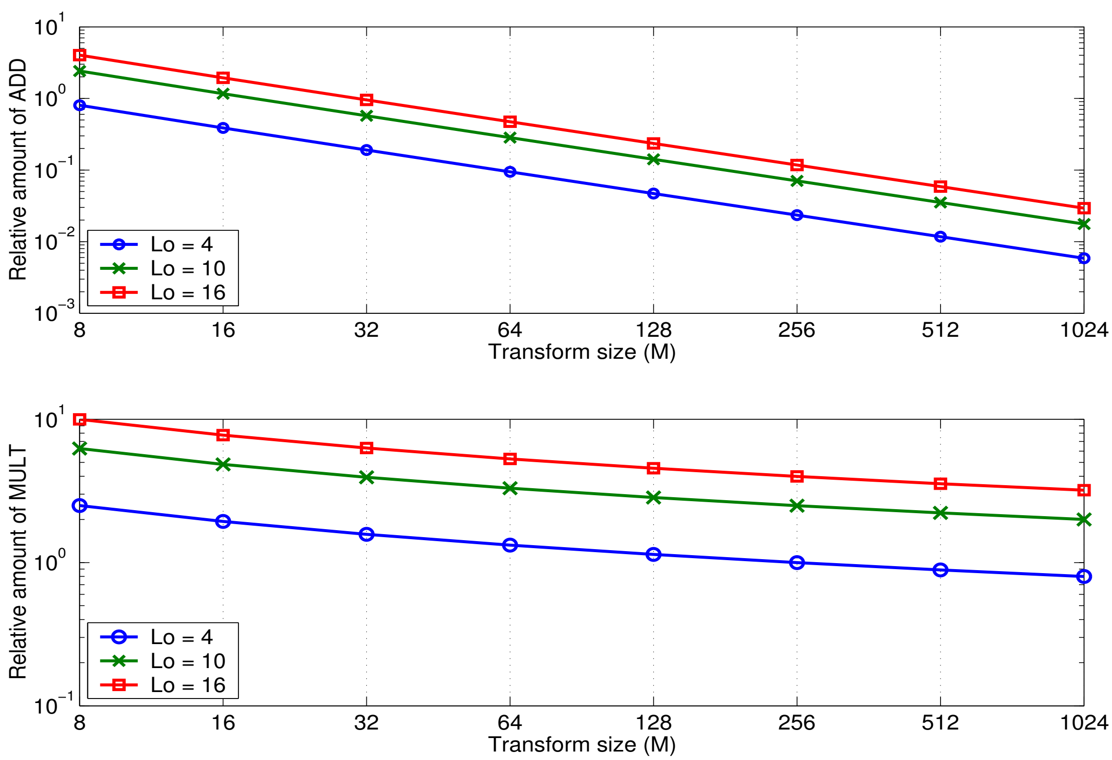

Fig. 13. Number of additions and multiplications required by the WPT relatively to the DFT as a function of the transform size M, for different wavelet generating filter lengths L_0.

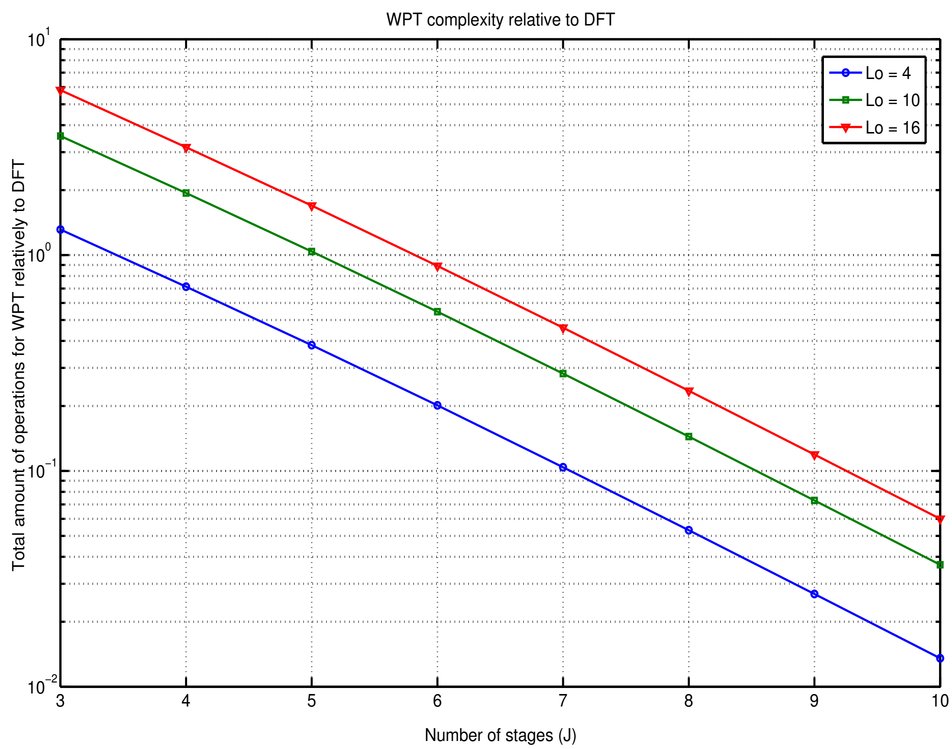

Fig. 14. WPT complexity relative to that of the DFT as a function of the transform size $M = 2^J$, for different wavelet generating filter lengths L_0.