Empirical Time and Frequency Domain Models of Spectrum Use

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Abstract

Dynamic spectrum access (DSA) has been proposed as a solution to the spectrum scarcity problem. However, the models for spectrum use, that are commonly used in DSA research, are either limited in scope or have not been validated against real-life measurement data. In this paper we introduce a flexible spectrum use model based on extensive measurement results that can be configured to represent various wireless systems. We show that spectrum use is clustered in the frequency domain and should be modelled in the time domain using geometric or lognormal distributions. In the latter case the probability of missed detection is significantly higher due to the heavy-tailed behaviour of the lognormal distribution. The listed model parameters enable accurate modelling of primary user spectrum use in time and frequency domain for future DSA studies. Additionally, they also provide a more empirical basis to develop regulatory or business models.

Key words: Wireless communications, Cognitive radio, Dynamic spectrum access, Spectrum measurements, Spectrum model, Time-frequency model

1. Introduction

The amount of traffic carried by wireless networks is constantly increasing due to rising number of users. Additionally, new commercially emerging applications often have higher data rate requirements. Although the efficiency of wireless transmissions has also been improved, e.g., by the application of adaptive multi-antenna and multi-carrier techniques, the demand for radio spectrum increases constantly. However, the allocation of spectrum bands for new services or technologies is a slow and cumbersome process due to the multiple involved stakeholders with highly diverse interests. Furthermore, almost all spectrum bands with commercially attractive propagation characteristics have been allocated for existing services. In contrast, a multitude of measurement campaigns carried out at various locations all over the world has shown that spectrum is underutilised in time and space [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12].

Dynamic spectrum access (DSA) has been proposed as a solution to the contradiction of high demand for additional spectrum and inefficient spectrum use under the current regulatory regime. Recently, DSA has attracted significant research work, see, e.g., [13, 14] and the references therein. One of the most popular DSA scenarios describes primary users, who hold official licenses to use a spectrum band, and secondary users, that opportunistically access that spectrum when it is not used by the primary users [15]. In this case, secondary users are obliged to ensure that no harmful interference is caused to the primary system. Secondary users perform spectrum sensing in order to identify unused spectrum bands, also referred to as spectrum white spaces, spectrum holes, or spectrum opportunities [13, 16]. Lately, increasing the efficiency of the spectrum sensing process in terms of a reduction of the false positive rate has been a topic of active research. Exploiting the statistics of spectrum use [17, 18, 19, 20] or possibly the deterministic behaviour

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of primary users [21, 22] have been proposed to make searching for spectrum white spaces more efficient. Another example of current research is the development of more flexible wireless systems that can also benefit from non-continuous spectrum holes. Adaptive multi-carrier techniques that do not use those subcarriers that would actively interfere with the primary transmissions are a promising approach in this area [23, 24, 25]. The evaluation of these examples and numerous other technologies requires good models for the spectrum use and quantitative understanding how it changes over time and frequency. However, although multiple spectrum measurement studies have been carried out by various research groups and under different spectrum regulations to the best of our knowledge none of these activities resulted in detailed and realistic models for spectrum use in more than a single spectrum band.

Following J. Mitola’s initial definition [26, 27], Cognitive Radios (CRs) are aware of their surroundings and adapt appropriately. Such devices need powerful and efficient online modelling capabilities to build up the required understanding of the environment. DSA-capabilities are also seen as the major advantage of CRs compared to legacy devices and spectrum models, that describe the radio environment, are one important building block for the CR-vision [28, 29, 30].

In this paper we introduce time-frequency models for spectrum use for various popular wireless services. The model parameters are extracted from measurement data collected through an extensive spectrum occupancy measurement campaign carried out in Germany and the Netherlands. We point out statistical characteristics of spectrum use in time and frequency that should be captured by good models. Namely, spectrum use is clustered in the frequency domain and can be accurately described using geometric and log-normal distributions in the time dimension. As an example of model applications we investigate the impact of the sensing duration on the sensing reliability taking advantage of our spectrum use model. The heavy-tailed behaviour of the lognormal distribution significantly increases the probability of missed detection and shows the importance of realistic spectrum use modelling. The presented model has various applications in analytical as well as simulation-based DSA research. Examples are the more accurate evaluation of spectrum sensing parameters or the evaluation of protocols for DSA networks.

The remainder of the paper is structured as follows. We review related work in Section 2 and describe our measurement setup in Section 3. Afterwards, in Sections 4 and 5, we discuss the statistical characteristics of spectrum use in the time and frequency domain. We validate our modelling approach based on selected wireless technologies in Section 6 and elaborate on some consequences of the model characteristics in Section 7. We analyse sensing reliability using our spectrum use model in Section 8 and conclude the paper in Section 9. Finally, we provide model parameters and guidelines how to generate artificial spectrum use data in the Appendix.

2. Related work

Spectrum use has been studied in several measurement campaigns, all showing a significant amount of underutilised spectrum [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12]. Giving detailed numbers for the amount of available spectrum is not reasonable because some systems, e.g., spread spectrum technologies, can hardly be detected with the commonly used measurement setups. However, even when including an additional margin to account for these shortcomings of the measurement setups, a considerable amount of unused channels has been identified. Furthermore, spectrum holes can also be found in extremely dense metropolitan areas such as New York City or Singapore [6, 11]. All campaigns carried out so far have been based on a spectrum analyser or a similar instrument as the main measurement device and can be further divided in two groups. The results in the first group have been gathered with very wideband setups that do not use technology-specific settings [2, 3, 4, 8, 9, 11]. The technology-specific results in the second group can be achieved by reconfiguration of the measurement setup before investigating the next spectrum band [1, 5, 6, 7, 10, 12]. The latter approach enables a more detailed evaluation of technology-specific aspects but the required reconfiguration limits the duration of each single measurement. Our campaign belongs to the first group due to our goal of performing very long measurements of up to two weeks.

Spectrum occupancy is usually characterised by the use of two main metrics. First, the power spectral density (PSD) is the received power over the resolution bandwidth. Second, the duty cycle (DC) describes the ratio of time during which the channel is declared as busy. In the measurement context, the occupancy
decision is usually done using energy detection [31] with a simple threshold since no detailed information on the detected signal is available. Noise uncertainty strictly limits the accuracy of these occupancy evaluations [32] and is another reason why exact numbers for the amount of vacant spectrum cannot be easily determined. We will limit our discussion in this paper to the case of binary occupancy since our model is not intended to reproduce different PSD levels and focuses on the binary information if a spectrum band is available for secondary use or not.

Although a reasonable number of measurement results exists, only limited work has been done in the area of spectrum modelling. Most of the related papers have been focused on other system aspects but the evaluation of the proposed enhancements requires a model for the underlying spectrum use. Due to the lack of detailed literature on spectrum use models, certain assumptions have been taken in order to provide a basis for protocol and algorithm evaluation. Datla et al. evaluated adaptive sensing using a uniform distribution for the duty cycle [17]. The busy durations of spectrum occupancy have been modelled using an exponential distribution. Starting from these assumptions they have shown that the sensing frequency should be lowered for channels with a high probability of being busy. In our earlier work we have discussed a modified beta distribution with explicit consideration of fully loaded and completely vacant channels as a good model for the distribution of the duty cycle over frequency [9].

In [18] Zhao et al. assume a first order Markov model for each channel resulting in geometrically distributed state holding times. Using this general model they develop an analytical framework that successfully considers aspects of spectrum sensing and medium access. In previous work [19, 20] we followed a similar approach by using a first order Markov chain for spectrum modelling. Additionally, we showed that nearly no clear periodicities can be identified in spectrum occupancy data.

Kim and Shin analyse the optimal selection of the sensing rate and the sensing order of channels in [33] using renewal theory. They provide near optimal results for exponentially distributed state holding times. In [34] Yang et al. follow a similar approach but apply proactive channel access. The authors use a similar analytical framework as developed in [33] to estimate in which channels the primary user will return with lowest probability. They propose to proactively switch the secondary communication to these channels and show that this approach helps to avoid disrupting the primary user and to maintain reliable communication between secondary users.

One major existing piece of work on spectrum modelling has been based on measurements of the industrial, scientific and medical (ISM)-band at 2.4 GHz using a vector signal analyser. The different hardware setup enables very high sampling rates and modelling of spectrum use on a symbol level. Geirhofer et al. [35, 36] evaluated the state holding times of vacant and busy states in the case of Wireless Local Area Network (WLAN) traffic. Due to the high sampling rate the complete Medium Access Control (MAC) behaviour can be identified in the trace and has been modelled appropriately. The result is a semi-Markov model with the two main states Transmit and Idle and additional substates to model the behaviour of the WLAN MAC protocol. The authors propose a hyper-Erlang fit for the state holding times of the Idle-state. Replacing the standard geometric distribution of Markov-models enables one to reproduce the difference between channel free periods caused by the exponential backoff applied in WLANs and the situations when no WLAN node has a frame to send.

Although we follow a similar modelling approach our measurements have been performed with a wideband spectrum analyser and a significantly lower sampling rate, which results in undersampling of the received signal. We cannot reproduce similar details of selected technologies but investigate several different technologies and consider much longer measurement durations. Collecting very long measurement traces enables us to select subtraces that represent the characteristics of the complete data set well. Additionally, model validation across different subtraces can easily be achieved. Considering multiple different systems allows us to develop a generic modelling methodology that can be applied to different technologies.

Finally, researchers have investigated spectrum use in the spatial dimension. There is rich and extensive literature on propagation modelling [37]. In the CR context radio environment maps have been introduced as promising concept for improved environment-awareness [28, 29]. Additionally, spatial statistics have been of specific interest as appropriate set of modelling tools for spatial spectrum models [38, 39, 30]. Spatial models are required mostly in the analysis of cooperative spectrum sensing [40, 41, 42, 43]. Here, we focus on the time and frequency domains and do not investigate spatial aspects.
Table 1: Spectrum analyser configuration used throughout the measurements.

<table>
<thead>
<tr>
<th>Centre frequency</th>
<th>Band 1: $f_c = 770$ MHz</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Band 2: $f_c = 2250$ MHz</td>
</tr>
<tr>
<td></td>
<td>Band 3: $f_c = 3750$ MHz</td>
</tr>
<tr>
<td></td>
<td>Band 4: $f_c = 5250$ MHz</td>
</tr>
<tr>
<td>Frequency span</td>
<td>1500 MHz</td>
</tr>
<tr>
<td>Resolution bandwidth</td>
<td>200 kHz</td>
</tr>
<tr>
<td>Number of measurement points</td>
<td>8192</td>
</tr>
<tr>
<td>Sweep time</td>
<td>1 s</td>
</tr>
<tr>
<td>Measurement duration</td>
<td>Up to 14 days per subband</td>
</tr>
<tr>
<td>Detector type</td>
<td>Average detector</td>
</tr>
<tr>
<td>Preamplifier</td>
<td>Up to 3 GHz: 28 dB gain</td>
</tr>
<tr>
<td></td>
<td>above 3 GHz: none or $\geq 24$ dB gain</td>
</tr>
</tbody>
</table>

3. Measurement setup

In this section we provide a concise description of our measurement setup and the selected measurement locations. We limit the discussion to the most important parameters and those details that have an impact on the modelling of spectrum use. Further implementation details have been presented in [8, 9].

We deployed an Agilent E4440A high performance spectrum analyser [44] together with a standard laptop in a weather-proof wooden but RF-shielded box. The laptop remote controlled the analyser via Ethernet using standard protocols for instrument control. We selected 200 kHz as the resolution bandwidth although several services use narrower channels. Such rather coarse frequency resolution enabled us to cover more bandwidth in one sweep and extend single measurements up to two weeks without the need to further extend the overall measurement time at one location. Due to the general trend in wireless communication towards more broadband technologies we also expect DSA systems to be broadband in nature. Therefore, also from an application point of view spectrum white spaces of few tens of kHz are not interesting for opportunistic use.

Although we investigated frequencies up to 6 GHz we focus here on the spectrum band from 20 MHz up to 3 GHz because most of the present popular wireless systems work in these spectrum bands. We used omnidirectional antennas and configured the sweep time to be 1 s. Taking into account additional delays for the data transmission between instrument and laptop and periodic instrument recalibrations, one week of measurement resulted in traces of about 335,000 sweeps (on average each sweep took 1.8 s or 1000 sweeps took about 30 min). The detailed set of measurement parameters is listed in Table 1.

In this paper we present results based on measurements taken at two different locations, one located in the Netherlands and one in Germany. We placed our measurement setup on the roof of the main building of the International School Maastricht. In the following we will refer to this location as NE. The view from the roof is open and the school is located in a residential area close to downtown Maastricht. The other location was selected as an example for locations where end-user devices capable of DSA may work in real home scenarios. We accordingly measured the spectrum use on the balcony on the third level of an older residential building in a central housing area of Aachen, in the following called AB.

The selection of a spectrum analyser as measurement instrument has certain consequences for the characteristics of the investigated primary user signals that can be captured. First, the swept analysis enables high dynamic range and sensitive power measurements in order to evaluate also very weak primary user signals. Second, it supports very wideband measurements that can evaluate multiple systems in parallel. Third, swept measurements do not support high sampling rates and the received signals are significantly...
undersampled. Fourth, since the pure signal power is recorded no signal characteristics can be investigated in order to differentiate between noise, interference, or primary user signals. Thus, no feature detection techniques can be applied [45]. Fifth, the swept method of operation and the low sampling rate inherently result in a system that may miss short primary user signals. Additionally, the detected bandwidth of wide-band signals may be too narrow if only short frames are transmitted. The spectrum analyser does not sweep through the complete signal bandwidth during the duration of the short frame.

Our goal was to evaluate the spectrum use in multiple spectrum bands in parallel and to collect spectrum data for measurement durations significantly longer than few hours. Both requirements limit the selection to a setup based on a spectrum analyser. In the following we will present modelling results for different technologies taking advantage of our wideband-capable measurement setup. However, the disadvantages of the generic setup without technology-specific parameter configurations prevents the reproduction of some of the details of selected technologies. We will point these aspects out where appropriate.

4. Statistical characteristics of spectrum use over time

As a basis for our modelling work we identified several statistical characteristics of spectrum use that should be reproduced by any realistic spectrum model. In this Section we present those aspects that describe the changes of spectrum use over time. In this context, we limit the evaluation to single measurement channels and build a time series of all samples taken at a certain frequency as part of consecutive sweeps. Based on our measurement configuration the sampling period is about 1.8 s. Several investigated time series have more than 685,000 samples collected throughout two weeks of measurements. Other measurements lasted for one week resulting in traces of about 335,000 samples.

Although the measurement results describe continuous PSD values we limit our modelling work to binary occupancy information. In DSA scenarios secondary users are mostly interested in the information if the primary user is present or not. Detailed signal strength data is less important. We applied energy detection based on the threshold of $γ = -107$ dBm as given for 200 kHz channels in an earlier requirements document of the IEEE 802.22 standardization committee [46, 47, 48]. Although initially specified, the selected threshold will not be applied for detection of wireless microphones by final IEEE 802.22 certified products because the Federal Communications Commission (FCC) requested to detect wireless microphones at similar PSD levels as applied for TV broadcasting stations [49]. However, our measurement setup is not sensitive enough to fulfill these stricter regulations and we stick to the initially proposed value. The presented problem formulation and modelling methodology is not specific to energy detection and is also applicable to occupancy information obtained from feature detectors or other types of spectrum sensors.

Let $Ω_{t,i}$ denote the spectrum occupancy at time index $t$ and channel index $i$, defined as

$$Ω_{t,i} = \begin{cases} 0 & \text{if } PSD_{rx,t,i} < γ \\ 1 & \text{if } PSD_{rx,t,i} \geq γ \end{cases}$$

where $PSD_{rx,t,i}$ is the received PSD measured in channel $i$ and at time index $t$. Note that the inter-sample time is not constant and the time index $t$ does not correspond exactly to the measurement time. If $Ω_{t,i} = 1$ a primary user signal is detected and the channel is called occupied. PSD values below $γ$ indicate a free channel, respectively. Furthermore, let us denote by $N_i$ the number of measured samples in channel $i$ and by $DC_i$ the duty cycle computed for channel $i$:

$$DC_i = \frac{\sum_{t=1}^{N_i} Ω_{t,i}}{N_i}.$$  \hspace{1cm} (2)

Let $DC_{m}^n$ denote the duty cycle in the channel $i$ evaluated over time period $m$ out of $M$ time periods. All time periods consecutively build up the complete measurement trace. In this Section we investigate the statistics of a single channel and can thus omit the channel index $i$, writing

$$DC_{m}^n = DC^m = \frac{\sum_{t=K_i,\ldots,m-1+K_m}^{K_i,\ldots,m-1+K_m} Ω_t}{K_m}, \quad m \in [1, \ldots, M]$$

(3)
where $K_m$ is the number of samples in the time period $m$ and $K_{1,...,m-1}$ denotes the number of samples during all previous time periods. We do not limit the number of samples per time period but limit the duration of each period, denoted by $T$, because the sampling rate is not constant as described above. Therefore, the number of samples measured during time period $m$, $K_m$, does vary with $m$ but the measurement duration $T$ of each period is the same. The numbers of samples $K_m$ measured during all $M$ time periods sum up to the number of all samples, that is, we have

$$\sum_{m=1}^{M} K_m = K_{1,...,M} = N.$$ (4)

Figure 1 shows an example how $DC^m$ changes with $m$ for time periods of one hour duration. We selected a channel allocated to the downlink (DL) of the Global System for Mobile Communication (GSM) in the version working in the 1800 MHz band. The channel carried an intermediate load that varied with the time of day. As expected, the 24 h cycle is clearly visible and also an additional cycle of one week length can be identified. Every weekday exhibits high usage around noon but the peak usage occurs around 20:00 in the evening. The network usage is higher during the week but also on Sunday afternoons and evenings significant network load has been detected. The graph indicates that taking the whole measurement trace as basis for spectrum modelling may not be appropriate and researchers should pay attention to this fact in the future.

For DSA applications it is important to model the durations of occupied and free periods realistically. We define a continuous period without a single free sample as a burst since the investigated channel must not be accessed opportunistically at any time. We call a period of consecutive free samples a run, respectively. Similar terminology and modelling methodology has been used to describe and model the binary error behaviour of wireless channels [50, 51, 52, 53, 54, 55]. Although the $DC$ may be same, a scenario with one very long run is more attractive for DSA applications than the case with several shorter runs. Exploiting several short runs without causing harmful interference to the primary system is significantly more difficult. The detection performance in the case of non-continuous primary user signals also heavily depends on
the run and burst length distributions. Therefore, reproducing not only the DC but also the lengths of runs and bursts is an important characteristic of a realistic spectrum use model. Additionally, we have to investigate if run and burst lengths can be assumed to be independent and identically distributed (i.i.d.) or if correlations between consecutive samples or specific periodicities exist. In the following we will at first evaluate possible correlations and periodicities in the run and burst lengths over time and, afterwards, investigate the distributions of runs and bursts.

4.1. Correlations and periodicities in run and burst lengths over time

Figures 2 and 3 show the correlation of the run and burst lengths over time based on another GSM1800DL-channel with intermediate load. We used the whole trace as input and, again, we can clearly identify the 24 h cycle. The weekly cycle is also present. The correlation will be higher if the lag corresponds to six or seven whole days compared to a lag of few full days when both weekend days are compared to weekdays. The difference between runs and bursts is caused by the duty cycle, which is below 50 % and results in, on average, longer runs than bursts. The correlation is very high for lags that roughly correspond to multiples of days. Therefore, modelling the length of consecutive runs and bursts as i.i.d. is not appropriate.

Accurate reproduction of the correlation between adjacent run and burst lengths and the present periodicities could be achieved by detailed time series analysis, e.g., by Auto Regressive Integrated Moving Average (ARIMA) models \[56, 57\]. However, we want to limit the number of required model parameters and investigate if shorter traces show similar correlations. Additionally, the traffic in secondary networks will vary with time of day most probably in a very similar way as in primary networks. Therefore, describing the spectrum use during the day will be sufficient for practical spectrum models.

Figure 4 shows a comparison of the correlation of the run lengths over time for different selected time periods. We selected a trace which describes the middle of a Digital Enhanced Cordless Telecommunications (DECT) channel. The dynamic channel selection (DCS) feature of DECT systems results in load balancing across all DECT channels and a more random behaviour in terms of spectrum use compared to other
Figure 3: Correlation of burst lengths over time based on a GSM1800DL-trace of two weeks length. The investigated channel carried an intermediate traffic load ($DC \approx 30\%$ during the whole measurement duration).

Figure 4: Correlation of run lengths over time based on a DECT-trace of selected time periods of different length. The investigated channel carried a typical traffic load for a DECT-channel ($DC \approx 50\%$).
technologies [58]. We compare time periods of duration $T = 4\,\text{h}$, $8\,\text{h}$, and $12\,\text{h}$. The $T = 4\,\text{h}$ period lasts from 12:00 to 16:00 and the $8\,\text{h}$ and the $12\,\text{h}$ time periods both start at 11:00 and cover most of the daytime and parts of the evening. When assessing the randomness of a measured parameter, a threshold is often applied to decide upon the significance of possibly detected correlations [59]. Let $\tau_T$ denote this threshold:

$$\tau_T = \frac{1.96}{\sqrt{R_{T,m}}}$$

where $R_{T,m}$ is the number of detected runs during the time period $m$. Depending on the type of spectrum use, $R$ may vary drastically between technologies. Very bursty traffic will result in fewer but longer runs and bursts. In contrast, more frequent but shorter runs and bursts are caused by uniformly distributed load over time. If the traffic type stays relatively constant, $R$ will increase with the evaluated time period $T$.

We have seen periodicities of 24 h and one week length. Further detailed study of our measurement data did not show additional periods. Therefore, selecting time periods shorter than one day, as shown in Figure 4, solves the problem of periodicities present in the data. However, correlations between consecutive run- and burst lengths may exist. In order to visualize these conditions more explicitly, Figure 4 shows only lags up to 45 samples. The correlations are lower compared to the results shown in Figures 2 and 3 indicating that the periodicities introduce stronger correlations than ones existing between consecutive run or burst lengths. The computed values are slightly above their significance thresholds only at very few lags. Investigating time periods shorter than one day also solves partially the issue of correlations between adjacent run and burst lengths. Additionally, a time period $T = 12\,\text{h}$ seems to be appropriate for spectrum modelling since a further reduction of the considered measurement duration does not lower the correlations between consecutive length samples but reduces the amount of data used as basis for further modelling steps.

These results apply for the DECT technology. Before deciding on the modelled time period we also evaluated a second example. Figure 5 shows a similar graph for a GSM1800DL channel with low traffic load (DC ≈ 14.4%).

![Figure 5: Correlation of run lengths over time based on a GSM1800DL-trace of selected time periods of different length. The investigated channel carried a low traffic load (DC ≈ 14.4%).](image-url)
load. Each run is significantly longer and the number of runs extracted per time period is lower giving a higher significance threshold per time period compared to the DECT case. These differences indicate the importance of which time period of a given length is extracted from the whole measurement trace for detailed study. We compared the run and burst length distributions computed for each time period to all other distributions based on time periods of same length. Afterwards, we chose the time period with the least difference to all other time periods starting at the same time of day.

GSM applies time division multiple access (TDMA) at the MAC-layer with short time slots of 0.577 ms. Our measurement setup evaluates each of the configured 8192 measurement channels of 200 kHz bandwidth for about 0.122 ms and thus much shorter time than a single GSM time slot. Although a GSM call lasts long enough to overlap multiple measurement sampling times of 1.8 s, the result does not have to be a burst of the same length as the call duration since the sweep may sometimes hit the used time slot but may sometimes also measure an unused time slot. This interaction of the GSM time slot structure and the low sampling rate applied in our measurement setup may result in the detected correlations between consecutive run and burst lengths. Therefore, the correlations that we detected in our traces do not exactly correspond to system characteristics because call durations are not perfectly captured. Additionally, the correlations shown in Figure 5 are exceptionally high due to the low load carried by the examined channel. The reader should note that single calls can significantly change the statistics of the spectrum use.

Although slight correlations are present in the data we will not model those in detail. Instead, we assume the run and burst lengths to be i.i.d., while allowing for different distributions for run and burst length distributions, respectively. However, a detailed study of the detected correlations with a specifically designed measurement setup with higher sampling rate is well motivated. Especially, the interaction of GSM time slot structure and sampling rate would deserve closer evaluation.

4.2. Run and burst length distributions

We have limited our model to binary occupancy information with the states occupied and free. Additionally, we have verified that we can assume run and burst lengths to be i.i.d. within certain accuracy limits. These prerequisites enable us to apply a two-state semi-Markov model as framework for a single channel. It combines the simplicity of a two-state model with the flexibility to describe the state holding times in each state with arbitrary distributions. As described above, the two-state semi-Markov model has been popular in different areas of wireless communications research. Researchers have applied it to describe the error behaviour of wireless channels [50, 51, 52, 53, 54, 55] but also to describe the spectrum use in WLAN systems [35, 36] and it is also referred to as alternating renewal process.

In order to fully describe a single channel we have to determine realistic state holding time distributions. In the literature the exponential [17] and its discrete analogue, the geometric distribution [18], have been proposed.

Figure 6 shows the run length distribution measured in a selected channel in the 2.4 GHz ISM-band and fits for three selected candidate distributions. The first one is based on simple Bernoulli experiments. In this model not only the run lengths are assumed to be i.i.d. but also all individual occupancy samples are modelled independently. The probability of being occupied is set to the measured DC. Additionally, we consider the geometric distribution as result of a typical two-state Markov-model. The third candidate is the lognormal distribution, which we selected as an example for distributions that may be heavy-tailed based on their parameter configuration.

Figure 6 also lists several goodness-of-fit metrics that we use to compare the different distributions. The DC is a very straightforward metric and close reproduction is a minimum requirement for any spectrum model. We determined the DC analytically as the ratio of the expectation value of the burst length and the sum of the expectation values of run and burst lengths for each distribution. It depends on the accurate reproduction of both the run and burst length distributions.

Often the Kullback-Leibler divergence

\[ D_{KL}(f \mid \mid g) = \sum_k f(k) \log \frac{f(k)}{g(k)}, \]

is used to measure the difference between two probability distributions. In our case, we use it to compare the observed distribution with the theoretical distribution.

The Kullback-Leibler divergence is given by

\[ D_{KL}(f \mid \mid g) = \sum_{k=1}^{K} f(k) \log \frac{f(k)}{g(k)}, \]

where \( f(k) \) is the observed probability of observing \( k \) and \( g(k) \) is the expected probability of observing \( k \). This divergence is a non-negative number, and it is zero if and only if \( f = g \).
is used to evaluate the goodness of a distribution fit, where \( f(k) \) and \( g(k) \) are the two discrete probability density functions (PDFs) to be compared. Here, we use the symmetric version of the Kullback-Leibler divergence

\[
D_{KLsym}(f, g) = D_{KL}(f \| g) + D_{KL}(g \| f),
\]

in the Figures simply referred to as \( KL \). We do not solely rely on the symmetric Kullback-Leibler divergence because it requires accurate estimation of the PDFs and does not compare the cumulative distribution functions (CDFs) directly. Although we estimate the PDF using robust kernel-based methods the \( KL \)-metric is still prone to give misleading results, e.g., in the case of PDFs with narrow and high peaks.

We also apply an area-metric \( A \) as given, e.g., in [53]. It is defined as

\[
A(F, G) = \frac{1}{J} \sum_{j=1}^{J} \left| \log(F^{-1}(j/J)) - \log(G^{-1}(j/J)) \right| - \frac{\log(F^{-1}(1/J)) - \log(G^{-1}(1/J))}{2J} - \frac{\log(F^{-1}(1)) - \log(G^{-1}(1))}{2J},
\]

where \( F \) is the measured CDF and \( G \) is the CDF of the fitted model. The variable \( J \) denotes the maximum measured run or burst length since \( G \) can be determined for any arbitrary run or burst length. The inverse function \( F^{-1} \) for a stepwise defined distribution function \( F \) can be defined as

\[
F^{-1}(k) = \inf \{ q : F(q) > k \}.
\]

The inverse function \( G^{-1} \) is defined similarly.

Finally, we compute the weighted Kolmogorov-Smirnov (\( KS_w \)) metric as applied, e.g., in [60]:

\[
KS_w(k) = \frac{|F(k) - G(k)|}{\sqrt{G(k)(1-G(k))}}.
\]
We use the $K_{SW}$-metric because it is reweighted to consider also the differences at the extreme ranges of $k$ appropriately.

Figure 6 shows that the chosen example can be well fitted with a geometric distribution. Also the Bernoulli-case is acceptable but the $KL$-metric indicates a slightly worse fit. The lognormal fit is worse for all three considered metrics. The chosen example is taken from the ISM-band, which is used by several wireless systems. However, our measurement results, perhaps not surprisingly, indicate that WLAN was the dominantly used technology. WLAN is based on a distributed MAC-protocol with random backoff times and the access point (AP) usually sends a beacon message every 100 ms. Both facts lower the probability of very long runs. Other technologies with random behaviour such as the mentioned DECT systems with their DCS feature also result in geometric distributions for the run and burst length distributions. Additionally, if the statistics of runs and bursts look similar and the $DC$ is roughly around 50% also systems with deterministic MAC-layers such as GSM result in geometric distributions for both run and burst lengths.

In [35, 36] Geirhofer et al. presented a hyper-Erlang distribution as a good fit to the idle times of WLAN channels. Although we concluded that WLAN was the dominant type of use of the 2.4 GHz ISM-band during our measurements, we do not evaluate the hyper-Erlang distribution as another option for distribution fitting. The sampling rate in our setup is significantly lower than the rate achieved in the vector signal analyser-based system deployed in [35, 36] and our measurement setup cannot capture the details that motivated Geirhofer et al. to apply the hyper-Erlang distribution. However, we successfully fitted geometric distributions to various different systems showing that the model has superior generality while accepting a lower accuracy in each technology-specific case.

The run and burst length distributions depend also strongly on the time of day. Figure 7 compares the run length distributions taken during different 12 h time periods. The day times start at 11:00 and the night times start at 23:00. During the day times the $DC \approx 50\%$ and the run length distributions can be fit well by geometric distributions. However, during the night times the load of the network significantly decreases and the probability for long runs increases. For these cases the assumption of geometrically distributed run
or burst lengths is no longer valid.

Figure 7 also shows an example for our approach of selecting a representative example trace out of the whole set of 12 h traces all starting at the same day time. The specifically marked selection is in the middle of all other distributions for the most important fraction of the CDF. Only at probabilities below $\approx 2\%$ it differs more from some of the other traces.

The differences between day and night times are very clear in the shown example. We selected this data set for visualisation reasons. In other data sets, again, especially in the case of very low or very high traffic load the two groups of distributions partially overlap because the maximum run or burst length may be significantly shorter and the differences between single traces further increase. Additionally, Figure 7 shows that even in the selected case with limited variations between days the distributions do change. Therefore, we do not fit specific features of selected traces since those will vary between measurement days and channels.

Figure 8 shows another example for a non geometric burst length distribution. The trace has been gathered in an uplink channel of GSM1800 with high load. All goodness-of-fit metrics indicate a better fit for the heavy-tailed lognormal distribution$^1$.

We found similar heavy-tailed behaviour in several of our traces. Typical cases are the run length distributions in channels with low load and the burst length distributions in channels with high load. This seems reasonable because in low load conditions fewer frames are transmitted and the probability of long runs increases. Similar arguments apply for bursts in high load scenarios.

The corresponding distributions of burst lengths in the cases of low load and small run lengths in nearly fully loaded channels are difficult to fit since the maximum run or burst length is usually only a few samples. However, the short maximum run and burst lengths in these cases also have significantly less impact on the overall spectrum use statistics. Therefore, the reproduction of the heavy-tailed behaviour using the lognormal distribution for the dominant lengths is sufficient to develop a realistic model.

$^1$In order to show the geometric distribution as straight line we have selected the semilogarithmic scale for Figure 6. In Figures 7 and 8 we have switched to the double logarithmic scale as usually used for run and burst length distributions.
The discussed examples clearly show that assuming geometric distributions for run and burst lengths does not describe all possible cases. Runs in low load scenarios are of special interest in DSA-scenarios since they describe valuable spectrum opportunities of increased duration and should be reproduced accurately by a realistic spectrum model.

5. Statistical characteristics of spectrum use over frequency

In the previous section we have discussed various aspects of modelling spectrum use over time. Next we extend our analysis by taking into account also the frequency dimension. We discuss possible correlations in the statistics over time between channels adjacent in frequency domain and introduce our approach how to accurately reproduce them.

Before giving examples from our measurement data we want to elaborate on the meaning of a channel in different wireless technologies. The interpretations are different and have significant impact on the question if we expect correlations between adjacent measurement channels.

First, we differentiate between measurement and technology-specific frequency channels. The measurement bandwidth is defined by the configured resolution bandwidth; in our measurements it was always set to 200kHz. The technology-specific bandwidth of a frequency channel is specified in the standard describing a certain technology. In the case of wideband signals, such as DECT \((B \approx 1.7 \text{ MHz})\) or the Universal Mobile Telecommunications System (UMTS, \(B = 5 \text{ MHz}\)), a single technology-specific frequency channel typically covers multiple measurement channels. In such scenarios very similar behaviour over time is to be expected for measurement channels that are part of the same primary user wideband frequency channel. In these cases the spectrum model should simply describe the statistics of a single technology-specific frequency channel including its bandwidth instead of modelling the correlations between multiple measurement channels of the configured resolution bandwidth.

Our measurement results confirm these findings although the correlations are lower than expected due to the swept analysis that rarely captures the full bandwidth of a pulsed signal. Further statistical analysis consistent with these results is given in [61].

Second, we examine correlations between technology-specific frequency channels. DECT uses time division multiplexing (TDM) with a fixed frame structure to allocate up- and downlink to the same frequency. However, the added DCS feature spreads the load over all available ten frequency channels. Due to the randomness of the frequency channel selection we do not expect any correlation in the time structure of the occupancy detected at adjacent DECT channels.

UMTS applies code division multiple access (CDMA) and more than a single frequency channel is rarely used. Each base station continuously announces its presence and at minimum one downlink code will always be in use. Thus, the downlink spectrum will never be vacant. In the uplink, multiple mobile nodes share the same bandwidth and rarely more than one 5 MHz channel is allocated. In our measurements the UMTS uplink use was too low to enable truly realistic modelling. Additionally, UMTS uplink signals are very weak due to the applied signal spreading and are difficult to detect with a spectrum analyser at large distances from the mobile transmitters.

The GSM system is the most interesting case in the frequency domain because it uses multiple narrow 200kHz channels in parallel to support a large number of users. Each mobile operator possesses a license for several GSM channels and can solely decide upon its allocation to different cells. Additionally, the allocation of users to frequency channels is important. Operators may fill one channel after the other one but they could also equalize the load over all available channels. Additionally, they may allocate adjacent frequency channels to the same cell but may also allocate adjacent frequency channels to different cells.

Figure 9 shows the binary occupancy information for approximately 30 min of measurements in the complete GSM1800 downlink band. The shown data was collected during a Friday afternoon in August 2007 at the residential location AB. The DC is clearly correlated over frequency. The fully loaded areas, shown in black, are heavily clustered in the frequency dimension. Similarly, vacant channels, shown in white, also rarely occur alone. Finally, also subbands with a low load, e.g., \(f \approx 1821 \text{ MHz}\), and an intermediate load, e.g., \(f \approx 1839 \text{ MHz}\), appear in groups. Clearly, such behaviour is a statistical characteristic that
should be reproduced by spectrum models because it has direct impact on the amount of consecutive vacant spectrum that secondary users may find. Additionally, the clustered white spaces in cellular bands are a good indicator that spectrum sharing scenarios can also be applied to cellular networks, as proposed, e.g., in [62].

Figure 9 seems also to indicate that at the edge of fully loaded spectrum subbands there are often channels with an intermediate load. However, this result is most probably a measurement artifact caused by our specific measurement configuration. Both our measurement resolution bandwidth as well as the GSM channel bandwidth are 200 kHz. Since we use 8192 measurement channels to cover a span of 1.5 GHz adjacent measurement channels slightly overlap in order not to miss weak signals centred at the border between two measurement channels. Additionally, the centre frequencies of the GSM channels and our measurement channels are different. Therefore, the intermediate load at the border of fully loaded subbands is most probably caused by the variation in signal strength of a signal that is only partially covered by the measurement channel on the border. Thus, the behaviour at the borders of the consistently used subbands does not need to be reproduced by a spectrum model.

5.1. Combination of different load levels to complete spectrum use models

We have seen that the \( DC \) is clustered in the frequency domain. Additionally, we have seen that the \( DC \) is similar inside all different types of groups: groups with vacant channels and groups with low, intermediate or very high traffic load. In order to reproduce this behaviour we define a limited number of archetypes of spectrum use and model the \( DC \) distribution by a simple step function. The two obvious steps describe vacant and fully loaded channels. Here, we define a vacant channel as \( DC < 0.5 \% \) and a fully loaded channel as \( DC > 97.5 \% \). We select an arbitrary number of intermediate steps based on the \( DC \) distribution as measured for a certain detection threshold. In most cases we defined three additional steps: low, intermediate, and high load, giving us a set of five archetypes in total. However, especially if the \( DC \) is low for most channels less steps will be sufficient, this limiting the number of model parameters.
Figure 10: Spectrum use in the DECT-band recorded for about 30 min at AB. The occupancy was determined using a threshold of $\gamma = -107 \text{dBm}/200\text{kHz}$.

We select representative example traces for each archetype and technology and perform detailed distribution fitting as discussed in Section 4.2. In this approach different load levels are reproduced with different semi-Markov models.

In order to capture the $DC$ correlation over frequency, we interpret the vector of $DC$ values over frequency as another time series and round the continuous $DC$ values to the previously defined $DC$ steps. Next, we evaluate the distribution of the bandwidths of all subbands consisting only of measurement channels belonging to the same archetype. As the number of these subbands is limited the following distribution fitting does not provide highly reliable results but indicates that a lognormal fit is a good choice. Additionally, the datasets collected at AB and NE both resulted in lognormal fits for the different GSM up- and downlink bands and add further confidence to this result.

6. Model validation

We have discussed different requirements for a good spectrum use model in the previous two Sections. Now, we continue with the comparison of spectrum use plots created from measurement data and spectrum data, which were artificially generated using the above described model. A more detailed description of the steps taken during the data generation process and appropriate model parameters are given in the Appendix.

6.1. DECT

We discuss DECT as the first example. We described several technology specific features, most notably the random channel selection, above. Figure 10 shows a white space map based on measurement data collected during an arbitrarily selected Saturday afternoon at AB. Several system characteristics can easily be seen. First, all ten specified channels can be identified. Second, the random load balancing over all ten channels is also obvious because no channel shows clearly more use than the others. Third, the characteristics
of the swept analysis are shown since single sweeps do not cover the full frequency channel bandwidth $B \approx 1.7$ MHz but the bandwidth can be identified when taking into account multiple consecutive sweeps. Fourth, the use seems to be higher in the middle of each channel. This impression is caused by two facts. DECT applies frequency modulation with $\Delta f = \pm 288$ kHz, which naturally results in lower load at the upper and lower end of the frequency channel bandwidth [58]. Additionally, a DECT transmitter has to fulfill a well defined spectrum mask and the transmitted signal is slightly attenuated towards its borders.

When generating artificial spectrum data we do not want to reproduce all of these attributes. The first two characteristics should be accurately modelled. However, the impact of the swept analysis, the modulation, and the spectrum mask should not be captured. Our sampling rate is by far too low to describe any modulation structure and a sensitive enough spectrum sensor would successfully detect the complete signal bandwidth.

As discussed in Section 5 we model technology-specific frequency channels. Additionally, no correlations between these channels have to be modelled due to the random nature of the DECT DCS feature. Figure 11 shows an example for an artificially generated white space map for the DECT band. The shortcomings of the measurement setup have been compensated for but the determined geometric run and burst length distributions have been accurately reproduced.

6.2. GSM1800 downlink

In the next step we evaluate the GSM1800DL case as an example that requires modelling of DC correlations over frequency. Figure 9 is an example of a measured white space map and we discussed its attributes in Section 5.

Figure 12 is an artificially generated example that accurately reproduces the characteristics of single channels over time. Additionally, the probabilities of each archetype, modelled by the heights in the DC step function, are captured by the underlying model. However, the DC correlation over frequency is neglected and the result is significantly different from the measured example.
Figure 12: Artificially generated spectrum use data based on the statistics extracted from measurement data taken in the GSM1800DL-band recorded at AB without taking into account the correlation between different channels.

Figure 13: Artificially generated spectrum use data based on the statistics extracted from measurement data taken in the GSM1800DL-band recorded at AB while taking into account the correlation between different channels.
Figure 14 shows the white space map as extracted from approximately 30 min of measurement data taken during a Thursday afternoon at AB. The three most popular WLAN channels, one, six, and eleven, centred at $f_1 = 2412$ MHz, $f_6 = 2437$ MHz, and $f_{11} = 2462$ MHz can be roughly identified with most traffic sent over channel one. Additionally, emissions from several other sources have been detected. The nearly continuous transmission at $f \approx 2410$ MHz cannot be unambiguously identified. Possible technologies are analog audio or video transmission links. Further WLANs configured to other than the three non-overlapping channels

In Figure 13 we present an example that applies the discussed time series modelling also to the DC vector over frequency. The bandwidth of each group of frequency channels, that all belong to the same spectrum use archetype, is sampled from the fitted lognormal distribution. Afterwards, the runs and bursts over time for each frequency channel are sampled from the fitted semi-Markov model as defined for the appropriate archetype. The visual comparison between Figures 9 and 13 confirms that our model can capture the main characteristics of the GSM1800DL spectrum use both in time as well as frequency domain. The major difference is the behaviour at the borders of the subbands of consistent spectrum use that was not modelled on purpose as explained in Section 5.

Detailed statistical comparison between the measured and artificially generated spectrum data confirms that the DC, the run and burst length distributions, as well as the correlation structure in frequency domain are accurately reproduced. The switch from measurement channels to non-overlapping frequency channels does also not cause any model inaccuracies validating the suitability of our generic measurement setup.

6.3. 2.4 GHz ISM-band

As third example we selected the 2.4 GHz ISM-band due to several reasons. It is one of the most heavily used bands and popular technologies such as WLANs and Bluetooth work in this band. It is also one of the rare examples with multiple services sharing the same spectrum band. Finally, due to the license-exempt regulatory model for ISM-bands it is well suited for testbed experiments evaluating DSA techniques and systems.

Figure 14: Spectrum use in the 2.4 GHz ISM-band recorded for about 30 min at AB. The occupancy was determined using a threshold of $\gamma = -107$ dBm/200 kHz.
and Bluetooth transmitters may also be present in the vicinity. The period of increased traffic near the end of the shown measurement excerpt is another artifact that cannot be fully explained. It may be caused by a microwave oven operated in a nearby kitchen. All these characteristics make the ISM-band an especially complex case for spectrum modelling.

Figure 15 shows an artificially generated white space map. It shows the limits of our modelling approach because the visual comparison between the artificial and the measured white space maps identifies several differences. The behaviour over time in single channels is reproduced well. However, the complex structure in frequency domain is not captured accurately.

The straightforward approach of limiting the modelling to WLAN activity and frequency channels of \( B = 20 \text{ MHz} \) to accurately reproduce their behaviour is also suboptimal due to multiple reasons. First, the complete WLAN traffic could not be captured using only three non-overlapping channels due to the presence of WLANs configured to other channels. Second, heterogeneity is a major characteristic of the ISM-band and this fact should be captured by an appropriate model. Third, the narrowband signal with high duty cycle that is reproduced by our model would not occur in the WLAN-focused approach. Fourth, the wideband signal detected at the end of the shown measurement period would also be missing.

The approach presented in this paper distinguishes time and frequency aspects and models both in separate steps. The applied divide-and-conquer modelling strategy is simpler than integrated models and enables direct understanding of the modelled phenomena. However, complex spectrum use statistics as found in the 2.4 GHz ISM-band are examples that seem to require an integrated approach for realistic time-frequency modelling.

7. Discussion

DSA-capable nodes are expected to be frequency-agile systems that can benefit from spectrum holes in a very wide spectrum band. However, reliable wideband spectrum sensing is a complicated process that
takes time and energy. If nodes do not have two radio interfaces the data transmission has to be stopped for spectrum sensing further underlining the importance of limiting the sensing time.

The modelling results presented in the previous Sections give further insight into the improvements that could be achieved when investigating deterministic primary user behaviour. We have seen that several systems result in geometric run and burst length distributions due to inherent system parameters. Additionally, run and burst lengths can often assumed to be i.i.d. Specifically designed aspects in MAC protocols or channel selection algorithms randomize the spectrum use and make the estimation of usage patterns very complex. Efficient forecasting of the primary user behaviour for more efficient spectrum sensing or even probabilistic access to spectrum bands, which have been estimated to be vacant, seems unrealistic in these cases.

Also in more deterministic systems, such as GSM, we have found geometric distributions and i.i.d. run and burst lengths in channels with intermediate load. The low sampling rate does not allow one to identify and extract the specified frame structure making spectrum use prediction nearly impossible. However, increasing the sensing rate is certainly a solution to this problem but it also significantly increases the involved overhead.

Opportunistic access to spectrum bands allocated to popular services such as GSM or DECT is certainly unrealistic at present. However, the interworking of devices that comply to different standard versions or even use completely different technologies gets more and more common in the wireless world. The inter-system interference problems in the 2.4 GHz ISM-band between, e.g., WLAN, Bluetooth, and sensor devices are only first examples [63, 64]. The ability to avoid interference in such heterogeneous environments to large extent is certainly an important feature of next generation systems independently of their DSA-capabilities. Additionally, in our measurement data the dominant fraction of services shows very simple always on or always off behaviour. Throughout our data analysis we have detected only few systems with duty cycles that are not close to one of these extremes. The limited availability of detailed tables on spectrum regulations and system descriptions for less popular technologies further lowers the number of candidate technologies for detailed evaluation.

However, we have also seen that correlations between adjacent run lengths may occur in channels with low traffic load. At the same time, channels exhibiting a low DC are very attractive candidates for opportunistic use. The detected correlations represent deterministic behaviour that could be exploited and is even present in popular commercial systems. Enhanced measurement methodology is required to estimate the potential gain in these cases. A simple increase in the sampling rate is most probably not enough to fully explore the primary user behaviour over time. Instead, coordinated evaluation on multiple layers may be a promising option. Several measurement campaigns have evaluated spectrum use and also the performance evaluation of commercial wireless systems is obviously well developed. Base stations or APs support extensive logging functions that could be combined with the developed spectrum occupancy measurement methodology. Together both approaches would enable the validation of approaches that extract the primary user behaviour from spectrum measurement data. Such estimates could be easily confirmed or refuted from the higher layer and network viewpoints.

8. Impact of the sensing duration on the sensing performance

The performance of spectrum sensing is usually evaluated using the receiver operating characteristics (ROC) that describe the trade-off between the probability of false alarm \( \text{p}_\text{fa} \) and the probability of missed detection \( \text{p}_\text{md} \). If the sensor is too sensitive strong noise samples may trigger false alarms. Missed detections will occur if the sensor is not sensitive enough to detect weak primary user signals.

The sensing duration will be of importance if non-continuous signals have to be detected. Wireless microphone transmissions are at present the most important example for non-continuous primary user signals [65]. If the primary user has a very low DC a too short sensing duration may result in a missed detection. In the following we will apply the results of the modelling work presented above in order to analyse the impact of the sensing duration on the detection performance. We investigate this case as an example on how the introduced model can be used to gain further insight in the performance of DSA-capable systems.
We assume that the spectrum sensor applies a slotted time model and will decide at the beginning of each time slot if a channel is sensed. Here, we set the time slot length to our inter-sample time defined by our measurement setup. We further assume a perfect spectrum sensor. Thus, if the sensor decides to examine a selected channel the sensing result will always be correct. Since in this model \( p_{fa} = 0 \), we will concentrate on \( p_{md} \) and its dependence on the sensing duration.

Under these assumptions the performance of an individual spectrum sensing action will solely depend on the time when the action starts and the sensing duration. Let \( T_s \) denote the start time of the sensing action and \( \Delta T \geq 1 \) the discrete sensing duration. The start time \( T_s \) may occur during a burst or during a run. The former case directly results in a successful detection of the primary user signal. If the sensing starts during a run the success will depend on the comparison of the remaining run length and the sensing duration \( \Delta T \). If the latter one is longer the primary user signal will be detected. If the run lasts longer a missed detection will occur.

Problems similar to the above have been studied by the operations research community in the context of reliability analysis of one-unit systems [66]. Those systems may consist of multiple components but their operational state is seen collectively. If a failure occurs the whole system will be inoperative, i.e., in the failed state, and once repaired the system is returned to the operating state. Both states map to the occupied and the free state in opportunistic spectrum access, respectively. Additionally, the distributions of the time-to-failure and the time-to-repair also map directly to the run and burst length distributions. The reliability analysis defines the interval reliability, which is the probability that at a specified time the system is in the operating state and does continue to do so for at least a given interval. In our scenario, the specified time corresponds to \( T_s \), the interval is \( \Delta T \), and the interval reliability is \( p_{md} \).

The interval reliability obviously depends on the run and burst length distributions and an analytical solution is difficult to compute for most distributions. Barlow and Hunter give in [67] the solution for the exponential distribution, which is the continuous analogue for the geometric distribution. If \( X \) is an exponentially distributed random variable with parameter \( \lambda \), \( Y = \lfloor X \rfloor \) will be a geometrically distributed random variable with parameter \( p = 1 - e^{-\lambda} \), where \( \lfloor \cdot \rfloor \) describes the rounding to the largest integer value not greater than the argument.

Adapted to the terminology used in this paper, we can rewrite the analytical solution as

\[
p_{md}(T_s, \Delta T) = \frac{\lambda_R e^{-\lambda_R(T_s+\Delta T)}}{\lambda_R + \lambda_B} + \frac{\lambda_B e^{-\lambda_B \Delta T}}{\lambda_R + \lambda_B} + \frac{\lambda_B^2 (1 - e^{-\lambda_R T_s}) e^{-\lambda_B T_s} e^{-\lambda_B \Delta T}}{\lambda_B (\lambda_R + \lambda_B)},
\]

where \( \lambda_R \) is the parameter of the exponential distribution of the run lengths and \( \lambda_B \) is the parameter of the exponential distribution of the burst lengths. In order to limit the impact of the initial state of the evaluated channel we examine

\[
\lim_{T_s \to \infty} p_{md}(T_s, \Delta T) = \frac{\lambda_B e^{-\lambda_B \Delta T}}{\lambda_R + \lambda_B},
\]

which depends only on the sensing duration \( \Delta T \) and the parameters of the exponential distributions \( \lambda_R \) and \( \lambda_B \).

The generic solution for other distributions, that cannot be solved in most cases in closed form, is given in [67]. Here, we investigate \( p_{md} \) for other than the exponential distribution through simulations. For an artificially generated occupancy trace we can determine all samples that result in a missed detection for a given \( \Delta T \). The fraction of the number of these samples and the length of the test trace gives the simulated \( p_{md} \).

Figure 16 compares the analytical and simulated solution for the exponential distribution and the simulation results for the lognormal distribution. We ran 200 simulations for each distribution with each using a test trace of 250000 occupancy samples. The small standard deviations given in the plot show the robustness of the simulation approach. The small difference between analytical and simulated solutions for the exponential case is due to the approximation for \( T_s \to \infty \), which we applied in the evaluation of the analytical result.

The parameters used to generate the artificial test traces are given in Table 2. In the case of the exponential distribution we also list the corresponding parameter for the discrete geometric distribution in order to enable easy comparison to the model parameters given in the Appendix.
Figure 16: Comparison of the impact of $\Delta T$ on $p_{md}$ based on analytical as well as simulation results for different run and burst length distributions.

Table 2: Distributions and their parameters used during the evaluation of $p_{md}$.

<table>
<thead>
<tr>
<th>Distribution</th>
<th>State</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exponential</td>
<td>Free</td>
<td>$p_R = 0.15, \lambda_R = 0.16$</td>
</tr>
<tr>
<td></td>
<td>Occupied</td>
<td>$p_B = 0.3, \lambda_B = 0.36$</td>
</tr>
<tr>
<td>Lognormal</td>
<td>Free</td>
<td>$\mu_R = 1.2, \sigma_R = 1.4$</td>
</tr>
<tr>
<td></td>
<td>Occupied</td>
<td>$\mu_B = 1.0, \sigma_B = 1.0$</td>
</tr>
</tbody>
</table>
Figure 16 shows that $p_{md}$ is heavily dependent on the underlying distribution. In the case of the exponential distribution approximately 14 samples are enough to fulfill $p_{md} < 10\%$. Additionally, $p_{md}$ will be negligible if $\Delta T \geq 50$. In the case of the lognormal distribution $\Delta T = 50$ is hardly enough to fulfill $p_{md} < 10\%$ and $\Delta T = 140$ still results in $p_{md} \approx 3\%$. Therefore, the heavy-tailed behaviour of the lognormal distribution considerably lowers the reliability of the sensing process. Only approximately 0.4\% of the runs in the case of the lognormal distributions are longer than 140 samples but the heavy-tailed behaviour results in few very long runs that cause the non-negligible $p_{md}$.

The above example shows the importance of appropriate spectrum modelling for any DSA study. The introduced model has various further applications. There is the evaluation of further parameters of the sensing process, such as the sensing rate or the bandwidth covered. Additionally, the investigation of protocols for DSA networks, that handle, e.g., the spectrum management in the case of multiple co-located secondary networks, requires good spectrum models. Another example is to examine the reliability of secondary networks and their outage behaviour.

9. Conclusions

In this paper we have introduced a flexible spectrum model extracted from real-life measurement traces. We have discussed spectrum use in the time and frequency domain and have applied a divide-and-conquer approach to model both dimensions separately. The developed spectrum model reproduces most of the identified spectrum use characteristics. Our key findings are:

- In the time domain, primary user activity can be well fitted by geometric and lognormal distributions depending on the channel load.

- The lognormal distribution is appropriate to describe the run length distribution in channels with low traffic load and the burst length distribution in the high traffic load case.

- In most cases the durations of runs and bursts can be modelled as i.i.d. random variables.

- However, correlations between adjacent run and burst lengths may occur in channels with low traffic load.

- In the frequency domain, channels that show similar statistical characteristics over time are clustered.

- The heterogeneity of the systems allocated to the 2.4 GHz ISM-band results in a complex use pattern that can hardly be modelled separately in time and frequency domain. An integrated time-frequency modelling approach seems to be required.

- The heavy-tailed behaviour of the lognormal distribution in time domain has significant impact on the spectrum sensing reliability. In the case of non-continuous primary user signals few long runs can cause a significant probability of missed detection.

- The above example shows the importance of accurate spectrum models in DSA research.

The introduced spectrum use model can be freely configured using parameters extracted from extensive measurements in various spectrum bands. The model can be applied to test diverse aspects of DSA systems ranging from spectrum sensing performance to protocols for DSA networks. The empirically validated distributions provide also a valuable basis for theoretical work on DSA algorithms.

We plan to examine further parameters of DSA systems using our spectrum model as part of our future work. Additionally, we will investigate the correlations between adjacent run or burst lengths and their relation to the heavy-tailed behaviour of the underlying distributions. Finally, we are currently in the process of restructuring our measurement data for appropriate publication.
Acknowledgments

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Appendix: Model parameters

In order to enable researchers to efficiently use the introduced spectrum model we list all the required steps to generate artificial spectrum use data. Additionally, we provide model parameters for several wireless technologies. All data sets have been extracted from our extensive measurement results taken at two different locations.

The Tables 3 and 4 provide all the needed parameters to model spectrum use over time. For most technologies multiple data sets are given in order to enable modelling of different channel load characteristics. Additionally, the probability of each archetype is provided to describe the overall load of each spectrum band. For some technologies parameters are given for both measurement locations. For other technologies either the spectrum band was fully loaded or too little use could be determined at one of the locations. Nearly completely vacant spectrum bands cannot be modelled reliably. Furthermore, in some cases external interference can be identified due to spectral signal characteristics. For example, the DECT-band at NE is highly interfered by non-DECT transmitters. Therefore, we did not perform similar modelling as done based on the DECT measurement data gathered at AB.

When comparing the extracted model parameters and the measurement results between locations we can identify few interesting insights. First, the distributions used to model the different load levels are mostly the same at both locations. This fact confirms that the distribution type describes not only load-specific but also technology-specific characteristics. Second, the load in the GSM900 uplink band was higher at NE but the load in the ISM-band was higher at AB. The balcony location at AB was very close to several WLAN APs. In contrast, the rooftop location at NE has good overview over the surrounding area but fewer short-range transmitters, such as WLAN devices, were close by. Third, we received significantly more interference at the rooftop location at NE.

We have seen that runs tend to be heavy-tailed distributed in low load scenarios. The same may happen for bursts in high load cases. This result is confirmed by several data sets given in the Tables 3 and 4. The corresponding other distribution, e.g., for burst lengths in channels with low load, is often difficult to determine reliably due to the very short maximum length. Since only few different lengths have been detected, the differences between distributions do not have significant impact and a slightly suboptimal fit does not severely alter the model accuracy.

Table 5 lists information on how to model the frequency dimension for the selected technologies. In addition to the number of channels and the technology-specific frequency channel bandwidth, we also give the parameters for the lognormal distribution to implement the clustering of spectrum use archetypes across a spectrum band. In the case of DECT, individual channels are not correlated and can be modelled separately. For the 2.4 GHz ISM-band no dominant frequency channel bandwidth can be determined and we use the resolution bandwidth used during the measurements instead.

Finally, we list the required steps to generate artificial spectrum data considering both the time and frequency domains. The process generates groups of channels belonging to the same spectrum use archetype until the specified bandwidth is reached. During the generation it is ensured that no channel load level occurs significantly more often than specified by its probability in the Tables 3 and 4. For technologies with predefined channel structure such as DECT the process has to be adapted appropriately.
Table 3: Parameters per wireless technology and archetype to model spectrum use over time extracted from measurement results taken at AB.

<table>
<thead>
<tr>
<th>Technology</th>
<th>Location archetype</th>
<th>Burst Distribution Parameters</th>
<th>Probability Ramps</th>
<th>Measurement Speciation</th>
</tr>
</thead>
<tbody>
<tr>
<td>GSM900 uplink</td>
<td>AB</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Inertial</td>
<td>B</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$d = d^r, DC = 0.009, \phi = 0.01$</td>
<td>R</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>GEOMETRIC</td>
<td>B</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$d = d^r, DC = 0.009, \phi = 0.01$</td>
<td>R</td>
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<tr>
<td></td>
<td></td>
<td>GEOMETRIC</td>
<td>B</td>
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<tr>
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<td></td>
<td>$d = d^r, DC = 0.009, \phi = 0.01$</td>
<td>R</td>
<td></td>
</tr>
<tr>
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<td></td>
<td>GEOMETRIC</td>
<td>B</td>
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<td></td>
<td>$d = d^r, DC = 0.009, \phi = 0.01$</td>
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<td>ISM-band at 2.4 GHz downlink</td>
<td>AB</td>
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<td></td>
<td>Inertial</td>
<td>B</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$d = d^r, DC = 0.009, \phi = 0.01$</td>
<td>R</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>GEOMETRIC</td>
<td>B</td>
<td></td>
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<td></td>
<td></td>
<td>$d = d^r, DC = 0.009, \phi = 0.01$</td>
<td>R</td>
<td></td>
</tr>
<tr>
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<td></td>
<td>GEOMETRIC</td>
<td>B</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$d = d^r, DC = 0.009, \phi = 0.01$</td>
<td>R</td>
<td></td>
</tr>
<tr>
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<td></td>
<td>GEOMETRIC</td>
<td>B</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$d = d^r, DC = 0.009, \phi = 0.01$</td>
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<td></td>
</tr>
<tr>
<td>DECT AB standard</td>
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<td>-</td>
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</tr>
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<td></td>
<td></td>
<td>Inertial</td>
<td>B</td>
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<td></td>
<td>$d = d^r, DC = 0.009, \phi = 0.01$</td>
<td>R</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>GEOMETRIC</td>
<td>B</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$d = d^r, DC = 0.009, \phi = 0.01$</td>
<td>R</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>GEOMETRIC</td>
<td>B</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$d = d^r, DC = 0.009, \phi = 0.01$</td>
<td>R</td>
<td></td>
</tr>
</tbody>
</table>

Notes: Parameters per wireless technology and archetype to model spectrum use over time extracted from measurement results taken at AB.
### Table 4: Parameters per wireless technology and archetype to model spectrum use over time extracted from measurement results taken at NE.

<table>
<thead>
<tr>
<th>Wireless technology</th>
<th>Measurement location</th>
<th>Spectrum use archetype</th>
<th>Probability [%]</th>
<th>Runs/Bursts</th>
<th>Distribution</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>GSM900 uplink</strong></td>
<td>NE</td>
<td>Vacant</td>
<td>0.00</td>
<td>-</td>
<td>Geometric</td>
<td>DC = 0.24, g = 0.15</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Low</td>
<td>0.21</td>
<td>R</td>
<td>Geometric</td>
<td>p = 0.49</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Mid</td>
<td>0.26</td>
<td>R</td>
<td>Lognormal</td>
<td>DC = 0.46, μ = 0.75, σ = 0.74</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>B</td>
<td>Lognormal</td>
<td>μ = 0.59, σ = 0.70</td>
</tr>
<tr>
<td></td>
<td></td>
<td>High</td>
<td>0.26</td>
<td>R</td>
<td>Geometric</td>
<td>DC = 0.69, p = 0.59</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>B</td>
<td>Lognormal</td>
<td>μ = 0.92, σ = 0.85</td>
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<tr>
<td></td>
<td></td>
<td>Full</td>
<td>0.27</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td><strong>GSM1800 uplink</strong></td>
<td>NE</td>
<td>Vacant</td>
<td>0.32</td>
<td>-</td>
<td>-</td>
<td>-</td>
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<td></td>
<td></td>
<td>Low</td>
<td>0.53</td>
<td>R</td>
<td>Lognormal</td>
<td>DC = 0.06, μ = 2.26, σ = 1.18</td>
</tr>
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<td>0.04</td>
<td>R</td>
<td>Geometric</td>
<td>DC = 0.51, p = 0.45</td>
</tr>
<tr>
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<td></td>
<td></td>
<td>B</td>
<td>Geometric</td>
<td>p = 0.44</td>
</tr>
<tr>
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<td></td>
<td>High</td>
<td>0.05</td>
<td>R</td>
<td>Geometric</td>
<td>DC = 0.91, p = 0.74</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>B</td>
<td>Lognormal</td>
<td>μ = 1.63, σ = 1.29</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Full</td>
<td>0.05</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td><strong>ISM-band at 2.4 GHz</strong></td>
<td>NE</td>
<td>Vacant</td>
<td>0.19</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Low</td>
<td>0.55</td>
<td>R</td>
<td>Geometric</td>
<td>DC = 0.08, p = 0.08</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Mid</td>
<td>0.07</td>
<td>R</td>
<td>Geometric</td>
<td>DC = 0.40, p = 0.33</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>B</td>
<td>Geometric</td>
<td>p = 0.49</td>
</tr>
<tr>
<td></td>
<td></td>
<td>High</td>
<td>0.03</td>
<td>R</td>
<td>Geometric</td>
<td>DC = 0.82, p = 0.70</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>B</td>
<td>Lognormal</td>
<td>μ = 1.17, σ = 1.01</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Full</td>
<td>0.16</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
Table 5: Parameters per wireless technology to model spectrum use over frequency.

<table>
<thead>
<tr>
<th>Wireless technology</th>
<th>Measurement location</th>
<th>Number of channels</th>
<th>Channel bandwidth $B$ [MHz]</th>
<th>Center frequency $f_c$ [MHz]</th>
<th>Parameters of the lognormal group distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>GSM900 UL</td>
<td>AB</td>
<td>124</td>
<td>0.20</td>
<td>$890.2 + (k - 1)B$</td>
<td>$\mu = 1.2, \sigma = 0.9$</td>
</tr>
<tr>
<td>GSM1800 UL</td>
<td>AB</td>
<td>277</td>
<td>0.20</td>
<td>$1820.2 + (k - 1)B$</td>
<td>$\mu = 1.2, \sigma = 1.3$</td>
</tr>
<tr>
<td>DECT</td>
<td>AB</td>
<td>10</td>
<td>1.73</td>
<td>$1881.8 + (k - 1)B$</td>
<td>-</td>
</tr>
<tr>
<td>ISM (2.4 GHz)</td>
<td>AB</td>
<td>417</td>
<td>0.20</td>
<td>$2400.1 + (k - 1)B$</td>
<td>$\mu = 2.3, \sigma = 1.5$</td>
</tr>
<tr>
<td>GSM900 UL</td>
<td>NE</td>
<td>174</td>
<td>0.20</td>
<td>$880.2 + (k - 1)B$</td>
<td>$\mu = 0.8, \sigma = 0.7$</td>
</tr>
<tr>
<td>GSM1800 UL</td>
<td>NE</td>
<td>374</td>
<td>0.20</td>
<td>$1710.2 + (k - 1)B$</td>
<td>$\mu = 1.3, \sigma = 0.8$</td>
</tr>
<tr>
<td>ISM (2.4 GHz)</td>
<td>NE</td>
<td>417</td>
<td>0.20</td>
<td>$2400.1 + (k - 1)B$</td>
<td>$\mu = 1.6, \sigma = 1.3$</td>
</tr>
</tbody>
</table>

1. Select a technology as basis for the data generation and specify the bandwidth of the spectrum band for which artificial data is to be generated. Additionally, specify how many samples in the time domain shall be generated for each channel.

2. Use a uniformly distributed random variate and the probabilities per archetype to decide upon the spectrum use archetype of the new group of channels.

3. If it is not the first generated group check if it does belong to the same archetype as the previous group. If it does go back to the previous step and recompute the archetype of the new group until it differs from the archetype of the previous group.

4. Sample the number of channels that belong to the new group from the lognormal distribution described in Table 5.

5. Reduce the computed group width if the probability of the current archetype is much lower than the fraction of the cumulative bandwidth of all groups that belong to the current archetype and the overall spectrum bandwidth. This fraction should not be more than 10% higher than the listed probability of the current archetype.

6. Use the provided distribution parameters for the current archetype to model the behaviour over time of each channel in the new group. Each channel has to be modelled separately. The first state of the semi-Markov model of each channel can be determined using the listed $DC$ and a uniformly distributed random variate. Sample from both distributions in turn until the sum of the generated run and burst lengths is higher than the number of time samples specified in step 1.

7. Cut away unnecessary time samples.

8. The probabilities of the spectrum use archetypes should now be adapted before the type of the next group is computed. The more channels in previously generated groups belong to a certain archetype and the lower the initial probability of an archetype is, the lower should be the now updated probability of that archetype. These constantly updated probabilities should be used in step 2. In contrast, in step 5 the initial probabilities always have to be used to decide if a newly generated group adds too many additional channels that belong to its archetype.

9. Compare the sum of the number of channels in all groups to the number specified in step 1. If further channels are needed go back to step 2 and generate another group of channels.

10. Cut away unnecessary frequency channels.

References


27. Y. Zhao, L. Morales, J. Gaeddert, K. K. Bae, J.-S. Um, J. H. Reed, Applying radio environment maps to cognitive


