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Area Throughput and Energy Consumption for Clustered Wireless Sensor Networks

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***Abstract*—In this paper we present a mathematical approach to evaluate the *area throughput* and the energy consumption of a multi-sink Wireless Sensor Network (WSN). The WSN is organised into clusters, with one sink per cluster collecting data from sensors. A small variation of the Thomas point process is used to model sensors and sinks positions in the target area. We denote as *area throughput* the amount of samples per second successfully transmitted to the sinks. Both *area throughput* and energy consumption are strictly related to connectivity and MAC issues. The aim of this work is to devise a mathematical model that takes MAC and connectivity issues into account, under a common framework. We study the behavior of these two performance metrics when varying the *target rate*, defined as the maximum number of samples the network was deployed to deliver. Results show that a tradeoff between the *area throughput* and the energy consumption must be found. Finally, the impact of different sensors and sinks distributions on the *area throughput* is evaluated.**

I. Introduction

Wireless Sensor Networks (WSNs) [1] have become a subject of intense study. In the most common scenario considered in the literature a collection of sensor nodes (hereafter called simply *sensors*) is assumed to be deployed in a target area, each taking measurements on local environmental conditions which are then communicated to one or more sinks. Accurate estimation of the environment requires sufficient amount of measurement data sampled from the area of interest (target area). Because of this it is obviously of interest to study the capacity of such networks. Energy consumption is the other key performance figure for any WSN. The focus of this paper is to study both of these metrics in a unified framework.

When the number of sensors on the target area is large, several sinks are usually distributed to gather data and sensors are often further organised in clusters. One sink per cluster forwards the queries to sensors, and collects the responses. These networks, called *clustered* WSNs in the following, are precisely the ones studied here. The aim of this paper is to evaluate the *area throughput* and the energy consumed by the network in the above scenario. The *area throughput* is defined as the number of samples per unit of time successfully transmitted to the centralised unit, originating from the target area. Both the *area throughput* and energy consumption are analytically evaluated here and both MAC and connectivity issues are taken into account under a joint approach for the evaluation of these target metrics. All of the results are obtained as a function of the *target rate* defined as the maximum number of samples the network was deployed to deliver, that is, the amount of traffic that sensors are able to offer to the infrastructure. The analytical framework developed is general, but we also show how to apply it specifically in networks based on the widely adopted IEEE 802.15.4 MAC protocol [2]. We also explicitly consider the effect of sensors deployment. While uniform distribution of sensors after deployment is often a useful approximation, it is not always achievable or not even desirable in practise. For many deployment techniques, such as real drops, sensors tend to become placed in clusters of different sizes. The clustering of short range radios has been shown to occur also in other natural contexts, see, for example, [3]. We employ a parameterised model for describing these clusters, called the *Thomas point process* (TPP). The use of TPP allows us to characterise in detail the impact of the inhomogeneity and node densities on the metrics of interest. We shall give a detailed introduction to these models and other elements of our system model below.

The rest of the paper is organized as follows. We first give an overview of the state of the art in Section II. Section III introduces the system model and describes the considered deployment scenario. In Section IV the *area throughput* is defined and analytically evaluated and energy considerations are also provided. In Section V the mathematical model of 802.15.4 used here, is briefly introduced. Finally, in Sections VI and VII numerical results and conclusions are presented, respectively.

II. Related Works

Many works in the literature are related to the modeling of different MAC protocols, and also to connectivity models, but very few papers jointly consider the two issues under a mathematical approach. Some analysis of the two aspects are performed through simulations [4]. Many papers devoted their attention to connectivity issues of wireless ad-hoc and sensor networks in the past (e.g., [5]). Single-sink scenarios have attracted more attention so far. However, an example of multi-sink scenario can be found in [6]. The very typical models for nodes spatial distribution in static wireless networks (i.e., not considering distributions originated from a particular mobility

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pattern of nodes) are the Poisson point process (PPP) and the Binomial point process (BPP) with very few exceptions ([7] is one of them). In [8] and [9] the authors use (among others) a modified Thomas model [10] for describing real-world nodes deployments with a good accuracy. All the previously cited works do not account for MAC issues, with the one exception known to us in a slightly different context, namely the work by Hoydis et al. [11].

The model proposed here is based on the following previous works. The *area throughput* concept was introduced in [12] where authors also presented a mathematical model for its evaluation limited to the case of uniformly distributed sensors and sinks. Here we extend this analysis to non-uniform deployments. Moreover, in [12] no energy consideration was provided. The mathematical model used to derive the probability of success for the transmission of a packet in an IEEE 802.15.4 (Non Beacon-Enabled mode) single-sink scenario has been introduced in [13], [14]. Finally, in [13] a mathematical model to derive the energy consumption of an IEEE 802.15.4 device transmitting packets of a fixed size, is introduced. Here this model has been extended to handle the case where packets of arbitrary size (see Section V) are transmitted, and used for the evaluation of the energy consumed with respect to our reference scenario.

III. System Model

In this Section we detail the system model considered in the rest of the paper. We assume that sensors are deployed in clusters each containing one sink and a number of cluster members. We further assume that the sinks are deployed uniformly on a plane, with overall density ρ $[m^{-2}]$. For convenience in the calculations we assume an infinite plane surface on which sensors are deployed. In practise our results will be a good approximation in the finite area case provided the average transmission range of the devices is small enough. Otherwise, dealing with the edge effects would require additional work, and the shape of the deployment area should be included as a parameter in the system model. We then assume that each cluster associated to a given sink has a Poisson distributed number (with mean μ) of cluster members. The locations of these cluster members are taken to follow normal distribution with mean at the location of the sink, and with covariance matrix $\mathrm{diag}(\sigma_x^2, \sigma_y^2)$. This is a small variation of the TPP used as a sensor location distribution model in, for example, [9].

In order to reason about the connectivity structure of the clusters a model for connectivity is needed. We use the link model employed in [12], which accounts for both distance-dependent deterministic loss and channel fluctuations. It is given in terms of the connection probability for nodes at distance d apart, which takes the form

$$C(d) = 1 - \frac{1}{2}\mathrm{erfc}\left(\frac{L_{\mathrm{th}} - k_0 - k_1 \ln d}{2\sigma_S}\right), \qquad (1)$$

where σ_S is the standard deviation of the channel fluctuations Gaussian distributed, L_{th} is the maximum tolerated loss, k_0 and k_1 are model dependent constants.

For data gathering and communications we assume a simple polling model, in which the sinks periodically issue queries causing all the cluster members perform sensing and communicating their measurement results back to the sinks they are associated with. We also assume that for communication the sensors utilize standard IEEE 802.15.4 [2], with the sinks acting as PAN coordinators and network operating in the Non Beacon-Enabled mode. We assume that the different PANs established by the sinks use different frequency channels (spatial reuse is used in case more than 16 PANs are present). Therefore no collisions can occur between nodes belonging to different PANs. However, nodes belonging to the same cluster will compete on channel access when they try to transmit their packets to the sink. Since our focus is on gathering of data from sensors to sinks, we shall assume that any communication between the sinks does not interfere with the intra-cluster communications.

We shall conclude this section by a short discussion on the limitations and possible extensions of this system model. The assumption of IEEE 802.15.4 radio is in itself a non-controversial one. However, assuming the use of the IEEE 802.15.4 MAC layer is a choice that can obviously be relaxed. Numerous alternative MAC protocols have been developed by the sensor networking community, each with their own pros and cons. Nevertheless, due to its standardised state, we believe that the IEEE 802.15.4 MAC makes for a natural starting point. Also the network deployment model can obviously be changed. Such a change can be made in two manners: different sinks and sensors distributions could be studied and also the way sensors are associated to sinks can be modified.

IV. Derivation of Network Connectivity and Energy Consumption of Sensors

According to our assumptions, network connectivity is enhanced as the number of sensors that can gain access to a sink is made as large as possible. Communication from sensor to sink is permitted if the power received by the latter is sufficient (in which case the sensor is said to be *audible* to the sink), and if the number of (tentative) communication attempts taking place simultaneously is not too large (in which case we expect the transmission to be successful)[1]. The first condition relates to the physical layer, while the second is a MAC issue. Now we treat the two aspect separately in the next two subsections, ending up with a unified model that yields the *area throughput* [12], which is defined and characterized precisely in the following. Energy considerations are discussed in Section IV-C.

A. Evaluating Audibility of Sensors

Let us recall from our system model that we assume sinks uniformly distributed on the infinite plane with density ρ $[m^{-2}]$ and that sink each gives rise to a cluster which hence contains one sink and a number of cluster members, n, Poisson

[1]The reverse communication (sink to sensor(s)) only requires audibility, i.e., no MAC failures occur since different sinks use different frequencies.

distributed with mean μ. The p.d.f. of the positions of a sensor in a cluster is a 2D Gaussian, i.e.

$$f_{X,Y}(x,y) = \frac{1}{2\pi\sigma_x\sigma_y} e^{-\frac{x^2}{2\sigma_x^2}} e^{-\frac{y^2}{2\sigma_y^2}}, \tag{2}$$

where we assumed that the cluster center lies at the origin.

Now suppose each sensor has to reach its sink through direct single hop communication. If we employ the random connection model and denote by $C(d)$ the probability (1) that two sensors at distance d are audible, the probability that an arbitrary sensor in a cluster is audible to the sink is (after deconditioning with respect to the position)

$$p = \frac{1}{2\pi\sigma_x\sigma_y} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} C(\sqrt{x^2+y^2}) e^{-\frac{x^2}{2\sigma_x^2}} e^{-\frac{y^2}{2\sigma_y^2}} \, dx\, dy. \tag{3}$$

Assuming independence between two audibility events, we have for a single cluster

$$\text{Prob}\{k \text{ audible sensors}|n \text{ sensors in all}\} = \binom{n}{k} p^k (1-p)^{n-k}, \tag{4}$$

yielding

$$\text{Prob}\{k \text{ audible sensors}\} = \sum_{n=k}^{\infty} \binom{n}{k} p^k (1-p)^{n-k} \frac{\mu^n}{n!} e^{-\mu}. \tag{5}$$

The expected number of sensors per cluster that are audible to the sink is now given by

$$\bar{k} = \sum_{k=0}^{\infty} k \cdot \sum_{n=k}^{\infty} \binom{n}{k} p^k (1-p)^{n-k} \frac{\mu^n}{n!} e^{-\mu}. \tag{6}$$

B. MAC and Throughput Considerations

We assume that sinks periodically send queries to sensors belonging to their clusters and wait for replies. We denote by $f_q = 1/T_q$ the frequency of the queries transmitted by the sinks. Each sensor takes upon reception of a query one sample of a given phenomenon (e.g. atmospheric pressure, or temperature) and forwards it through a direct link to the sink. Once transmission is performed, it switches to an idle state until the next query is received. We call the interval between two successive queries a *round*.

We define the *area throughput*, denoted by S, as the number of samples per second successfully transmitted to the sinks from the target area A. Its derivation follows directly from the evaluation of the *cluster throughput*, S_c, defined as the number of samples per second successfully transmitted to a sink by the sensors belonging to its cluster.

By following the same rationale as in [12], we first consider the probability of successful data transmission by an arbitrary sensor to its cluster head, when n sensors are present in the cluster and k sensors out of n are audible to the sink (channel fluctuations are accounted for). This probability, $P_{s|n,k}$, can be computed as (from (3) and (4))

$$\begin{aligned} P_{s|n,k} &= p \cdot P_{\text{MAC}}(k) \cdot \text{Prob}\{k \text{ audible sensors}|n \text{ sensors in all}\} \\ &= p \cdot P_{\text{MAC}}(k) \cdot \binom{n}{k} p^k (1-p)^{n-k}, \end{aligned} \tag{7}$$

where we separated the impact of audibility and MAC on the transmission of samples (the sensor must be both audible to the sink and able to get its packet through). In particular, p is the probability that a randomly selected sensor in a cluster is audible to the sink (3), while $P_{\text{MAC}}(k)$ (with $k \geq 1$), is the probability of successful transmission when $k-1$ interfering sensors are present. This factor accounts for MAC issues and is derived for IEEE 802.15.4 in Section V.

Now for a cluster that has n sensors, and k of them are audible to the cluster head, we have for the cluster throughput

$$\begin{aligned} S_{c|n,k} &= n \cdot f_q \cdot P_{s|n,k} = n \cdot f_q \cdot p \cdot P_{\text{MAC}}(k) \\ &\quad \cdot \text{Prob}\{k \text{ audible sensors}|n \text{ sensors in all}\}. \end{aligned} \tag{8}$$

By first deconditioning (8) with respect to k we obtain

$$\begin{aligned} S_{c|n} &= n \cdot f_q \cdot p \cdot \frac{1}{M} \sum_{k=1}^{n} P_{\text{MAC}}(k) \\ &\quad \cdot \binom{n}{k} p^k (1-p)^{n-k} \text{ [samples/sec]}, \end{aligned} \tag{9}$$

which is the cluster throughput when n sensors are present in the cluster, with $M = \sum_{k=1}^{n} \binom{n}{k} p^k (1-p)^{n-k}$ being a normalizing factor. Recalling that $n \sim \text{Poisson}(\mu)$, we finally obtain

$$\begin{aligned} S_c &= f_q \cdot p \cdot \frac{1}{M} \sum_{n=1}^{+\infty} n \sum_{k=1}^{n} P_{\text{MAC}}(k) \\ &\quad \cdot \binom{n}{k} p^k (1-p)^{n-k} \frac{\mu^n}{n!} e^{-\mu} \text{ [samples/sec]}. \end{aligned} \tag{10}$$

Now note that in any closed domain of area A there are on average ρA clusters. For the sake of simplicity but without loss of generality we consider a square of side length L, so that $A = L^2$. Thus by assuming independence from cluster to cluster and neglecting border effects, i.e.,

- σ_x,σ_y small enough such that each cluster having its cluster-head in A is entirely contained in A with high probability;
- $L \gg$ average transmission range;

the *area throughput* S, is simply given by

$$\begin{aligned} S &= \rho \cdot A \cdot S_c = \rho \cdot A \cdot f_q \cdot p \cdot \frac{1}{M} \sum_{n=1}^{+\infty} n \sum_{k=1}^{n} P_{\text{MAC}}(k) \\ &\quad \cdot \binom{n}{k} p^k (1-p)^{n-k} \frac{\mu^n}{n!} e^{-\mu} \text{ [samples/sec]}. \end{aligned} \tag{11}$$

Now define the *target rate* G as the average number of data samples per unit of time the network was deployed to deliver. It is given by

$$G = \bar{N} \cdot f_q \text{ [samples/sec]}, \tag{12}$$

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where $\bar{N}$ is the average number of sensors in the selected area. By once again neglecting border effects (i.e. assuming that the border of the area does not cut off part of a cluster), the number of selected sensors is the product of two Poisson r.v.'s, namely the number of clusters times the number of sensors per cluster. As these numbers are uncorrelated, their expectations satisfy $\bar{N} = \rho A \cdot \mu$, from which

$$\mu = \frac{GT_q}{\rho A}. \quad (13)$$

Finally, by substitution of (13) into (11), we obtain

$$S(G) = \rho \cdot A \cdot f_q \cdot p \cdot \frac{1}{M} \sum_{n=1}^{+\infty} n \sum_{k=1}^{n} P_{\text{MAC}}(k) \cdot \binom{n}{k} p^k (1-p)^{n-k} \frac{\left(\frac{GT_q}{\rho A}\right)^n}{n!} e^{-\frac{GT_q}{\rho A}} \text{ [samples/sec]}. \quad (14)$$

C. Energy Considerations

Once a sensor receives the query coming from the sink it starts the algorithm to try to access the channel and, in case of success in accessing the channel, it transmits the packet. At the end of transmission, it switches off until the reception of the next query and in this state it does not consume energy. Therefore, a sensor consumes energy when receives the query and when it performs the MAC protocol (including states such as backoff, sensing, transmission, etc.). We denote as $E_{\text{MAC}}(k)$ the mean energy spent by a sensor for performing the MAC protocol. This energy depends on the mean number, k, of sensors audible to a sink and hence competing for the channel. Recall that $k \le n$ holds, where n is the number of sensors in the cluster. Obviously, in case a sensor is isolated (not audible by the sink) it will not spend energy for that round. Therefore, the mean energy spent by a sensor in the network in a round, E_{round}, is given by

$$E_{\text{round}} = p \cdot (E_{\text{rx}} + \overline{E_{\text{MAC}}}) \text{ [J/sample]}, \quad (15)$$

where p is given by (3), E_{rx} is the energy spent to receive the query and $\overline{E_{\text{MAC}}}$ is the mean energy spent for accessing the channel and transmitting the packet.

By following the same reasoning as before, for a cluster composed of n sensors we have

$$E_{\text{MAC}}|_n = \frac{1}{M} \sum_{k=1}^{n} E_{\text{MAC}}(k) \binom{n}{k} p^k (1-p)^{n-k}, \quad (16)$$

where we have averaged over the number of audible sensors (which are at most n) and $M = \sum_{k=1}^{n} \binom{n}{k} p^k (1-p)^{n-k}$. By further deconditioning with respect to n, we obtain

$$\overline{E_{\text{MAC}}} = \frac{1}{M} \sum_{n=1}^{+\infty} \sum_{k=1}^{n} E_{\text{MAC}}(k) \binom{n}{k} p^k (1-p)^{n-k} \frac{\mu^n}{n!} e^{-\mu}. \quad (17)$$

$E_{\text{MAC}}(k)$ is derived in Section V for the case of IEEE 802.15.4.

V. The Model of IEEE 802.15.4

In this Section we introduce the mathematical model used to derive the success probability, $P_{MAC}(k)$, together with the MAC energy consumption model when an IEEE 802.15.4 single-sink scenario is considered. The Non Beacon-Enabled mode of the IEEE 802.15.4 is assumed throughout [2]. The model used here has been initially developed in [13], [14]. For details on the protocol we refer to the standard as well. Here we simply underline that each time a sensor finds the channel busy it moves to a new backoff stage, trying again to access the channel. Since there exist $NB_{max}+1$ (where $NB_{max} = 4$) possible backoff stages, a maximum number of times a sensor can try to access the channel, is imposed. According to this, there will be a maximum delay affecting packet transmission.

We assume that nodes transmit packets of size $10 \cdot D$ Bytes, with D being a constant integer, and that the query has size 10 Bytes. Since a bit rate, R_b, of 250 kbit/sec is used, the time needed to transmit a packet is $D \cdot T$, where $T = 320\mu sec$, and the query is transmitted in T. In this case, the maximum delay a packet may experience in reaching the sink will be equal to $(121 + D) \cdot T$ [14]. Therefore, to ensure that all nodes have finished the CSMA/CA algorithm before the arrival of the next query, we set $T_q = (121 + D) \cdot T$.

The probability, $P_{\text{MAC}}(k)$, that a sensor successfully transmits its packet to the sink when $(k-1)$ interfering sensors are present is derived in [14].

Regarding energy consumption, in [13] the mean energy spent by a sensor when $(k-1)$ interfering sensors are present, $E_{\text{MAC}}(k)$, is derived, but only in the case $D = 1$. Here the extension of the model to the case $D > 1$ is provided.

According to the 802.15.4 CSMA/CA protocol, once the sensor receives the query, it starts the backoff algorithm, at the end of which it senses the channel. If the channel is found free the transmission occurs. Therefore, after reception of query, a sensor consumes energy when: (a) it performs backoff; (b) it senses the channel and (c) it transmits the packet. We let $P_s = 82.5$ mW be the power spent in receiving and sensing states; $P_{BO} = 50$ mW the power spent in backoff state and $P_t = 75.8$ mW the power spent in transmission (see Freescale IEEE 802.15.4 devices).

In the model time axis is divided into a finite number of slots of duration T and the probabilities of being in the different states depend on the slot considered. We denote by $P\{T^j\}$ the probability that a sensor finishes the transmission of a packet in slot j, and by $P\{S_k^j\}$ the probability of being in sensing in the kth backoff stage and at the jth slot. The mean energy spent by a sensor in a round is now

$$E_{\text{MAC}}(k) = \sum_{j=0}^{121+D} E_t^j(k) + E_s^j(k) + E_{BO}^j(k), \quad (18)$$

where E_t^j, E_s^j, and E_{BO}^j are the different energy contributions spent in transmission, sensing and backoff, respectively, for a sensor ending its transmission in slot j.

Since no retransmission is performed, each sensor will transmit only one packet per round. Therefore,

$$E_t^j = P_t \cdot \frac{D \cdot N_{bit}}{R_b} \cdot P\{T^j\}, \quad (19)$$

where $N_{bit} = 10$ Bytes is the number of bits transmitted in T. The energy spent in the sensing state depends on how many slots are used by the sensor for sensing the channel. A sensor transmitting in slot j could have sensed the channel for one slot, in case it has found the channel free at the end of the first backoff stage, for two slots in case it has found the channel free at the end of the second backoff stage, etc. This energy is given by

$$E_s^j = P_s \cdot \frac{N_{bit}}{R_b} \cdot (1 - p_b^{j-D}) \sum_{k=0}^{NB_{max}} (k+1) \cdot P\{S_k^{j-D}\}, \quad (20)$$

where p_b^j is the probability to find the channel busy in slot j, and $(1 - p_b^{j-D}) \cdot P\{S_k^{j-D}\}$ is the probability that a sensor at the end of the k-th backoff stage, finds the channel free and ends transmitting in slot j. Finally, the energy spent in the backoff state depends on how many slots are occupied by the sensor for the backoff procedure. This number depends, in turn, on the number backoff stages performed. Therefore, we have

$$E_{BO}^j = P_{BO} \cdot \frac{N_{bit}}{R_b} \cdot (1 - p_b^{j-D}) \sum_{k=0}^{NB_{max}} (j-k-D) \cdot P\{S_k^{j-D}\}, \quad (21)$$

where $j-k-D$ is the number of slots during which a sensor that has finished the k-th backoff stage, has performed backoff. This value is the same no matter what values of backoff counter are extracted in each backoff stage.

VI. Numerical Results

In this Section the behavior of the *area throughput* and of the energy consumption as functions of the *target rate*, G, for different packet sizes, clusters shaping factors and sink densities, are shown. In addition to illustrating results from the analytical calculations, we confirm these by showing results obtained from a simulator environment. Results are obtained by setting $A = 1\ km^2$, $k_0 = 40$ dB, $k_1 = 13.03$, $L_{\text{th}} = 95$ dB and $\sigma_S = 4$ dB. Moreover, in the following, we will assume $\sigma_x = \sigma_y = \sigma$. A square area, A, where sinks and sensors are distributed according to the small variation of the TPP described in Section III, is considered as target area.

In Figure 1, S as a function of G for different D, σ and sinks density, ρ, is given. Both analytical results (lines) and simulation results (markers) are shown. In the simulator clusters are formed in the following way: sensors choose the nearest, measured in Euclidean distance, sink to transmit to. Instead, the model forces a sensor to connect to the sink with respect to which it has been deployed according to the TPP (see Sec. III). As we can see a good agreement between results is obtained. The differences are due to border effects and the different cluster heads selection strategies. Of course, we expect that by increasing ρ and σ results will differ owing to the overlapping of clusters. As we can see, once we fix ρ and D, when the *target rate* is low, by decreasing σ, S gets larger; conversely, for high G, larger shaping factors improve performance due to fewer packet collisions. By increasing D and ρ, the intersections between curves related to $\sigma = 10$ [m] and $\sigma = 40$ [m] are obtained for lower values of G. In fact, once we fix ρ, an increase of σ brings to have a larger number of isolated nodes, but also to a smaller average number of sensors trying to connect to the same sink (i.e., fewer MAC losses). Therefore, for low G, connectivity is the main cause of losses and small σ are advantageous; conversely, for high *target rate*, it is better to fix large σ, to decrease MAC losses. Finally, we note that S shows a maximum: S increases with G till MAC losses become significant. The maxima are reached for larger values of G, when decreasing D and ρ.

In Figure 2 the energy per second per sample consumed (on average) by a single sensor in the network, $E = E_{\text{round}}/T_q$ [mJ/sec/sample] is shown as a function of G for different values of D and σ. The Figure was obtained by setting $\rho = 10^{-5}$ $[m^{-2}]$. As σ decreases, E gets larger since it is more likely that the sensor is audible to the sink and hence that it consumes energy at all. Moreover, for low *target rate*, by increasing D, E gets larger as well, since a greater amount of energy is spent for transmitting larger packets. Conversely, for high G, the larger D is, the lower will be the probability that a node succeeds in accessing the channel, decreasing the energy spent by the node.

By comparing Figures 1 and 2 we can deduce that a tradeoff between energy consumption and *area throughput* must be found.

In Figure 3 we show the behavior of $\eta = S/(T_q E_{\text{round}} G)$ [samples/sec/mJ], that is the number of samples per second received (on average) by the sinks, per mJ of energy spent. As expected, η increases by increasing ρ, since a greater number of sinks help reducing the size of clusters (thus reducing collisions and improving efficiency), and by decreasing D, since, once again, MAC losses decrease.

Finally, in Figure 4 we show $S(G)$ for different sensors and sinks distributions and demonstrate the impact of the cluster formation mechanism based on Monte Carlo simulations. The results clearly show the limitations on the area throughput imposed by fixed sink deployments, and the relatively good performance obtained by simple randomized cluster head selection. The feasibility of the latter approach is, however, clearly dependent on the application scenario considered.

VII. Conclusions

In this paper we studied the *area throughput* and the energy consumption of a *clustered* WSN. After defining the used system model we analytically derived the two performance metrics, obtained by combining the expressions for the connectivity structure of the network, with results on the operation of the IEEE 802.15.4 MAC layer. The developed framework is relatively general, allowing other sensor deployment strategies to be considered as well. The tradeoff between *area throughput* and energy consumption is shown. As part of

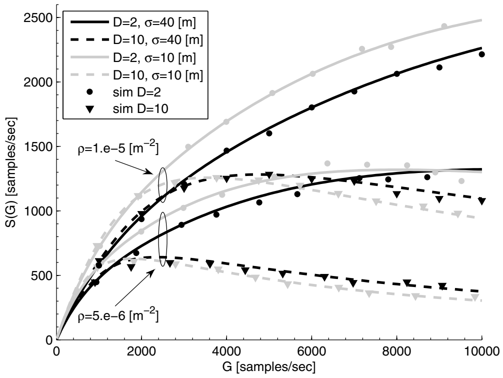

Fig. 1. Area throughput S as a function of G for different values of D, ρ and σ.

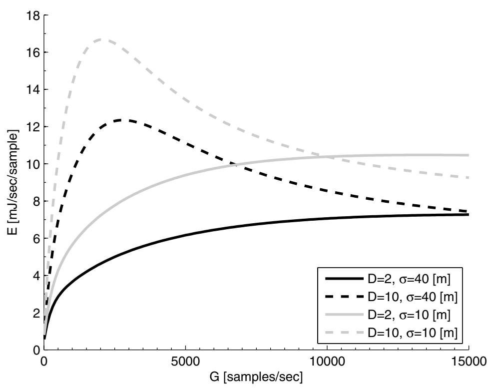

Fig. 2. E_{round}/T_q as a function of G, by varying D and σ.

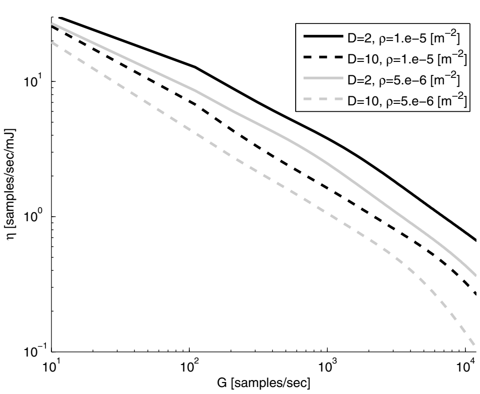

Fig. 3. η as a function of G, by varying D an ρ.

our future work we shall build on and generalise our present system model to allow, for example, for complicated cluster management protocols to be modelled and to cover periodic and event-based data acquisition as well. We believe that such methods turn out to be very important in studying the tradeoffs between different sensor deployment strategies in terms of sensing performance and energy consumption.

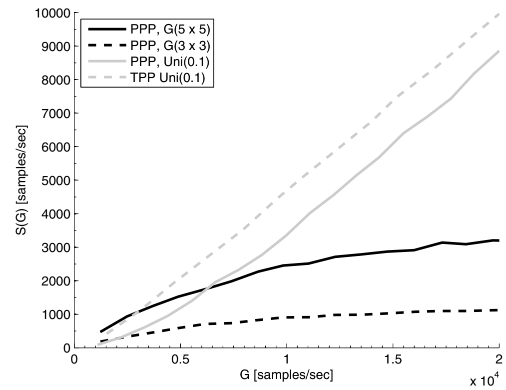

Fig. 4. The area throughput for different node location distributions (indicated in the legend) and cluster formation techniques ($G(n \times m)$ denotes cluster heads forming an $n \times m$ grid, and Uni(p) denotes unformly random selection of cluster heads from the node population with probability p).

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