Characterization and Modelling of Spectrum for Dynamic Spectrum Access with Spatial Statistics and Random Fields

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***Abstract*—There is need to develop better models and characterization methods for spectrum usage and radio environments of cognitive radios. Currently different theoretical and simulation based approaches towards enabling dynamic spectrum access would greatly benefit from the possibility to generate synthetic data for testing purposes. Such Radio Environment Maps must statistically exhibit the characteristics of realistic environments. Previous and on-going spectrum measurement campaigns are generating a vast amount of such data. In this paper we provide a partial answer to the spectrum modelling problem by showing how one can characterize and model spectrum maps with spatial statistics and random fields. We present the basic mathematical premises for building models and also through examples outline how one can generate useful statistics from real measurement data.**

I. Introduction

Dynamic spectrum access (DSA) [1] and characterization of the radio environment have become one of the most intensely studied aspects of cognitive radios and cognitive wireless networks. In particular cooperative spectrum sensing as possible improvement to local spectrum sensing has received significant attention (see, for example, [2]–[5]). Without cooperative sensing a single cognitive radio may miss a primary user transmission due to severe multipath fading or shadowing effects. The multipath fading characteristics experienced by cooperating cognitive radios will be correlated only over small distances in the order of the investigated wavelength [6]. However, the shadowing may be correlated over significantly further distances, e.g., if a group of cognitive users is located behind the same large building that severely attenuates the primary user transmission [7]–[11].

Until now most analytical approaches to cooperative sensing have been based on rather simple one-dimensional correlation models that consider only a single primary transmitter. These models will lead to the finding that cooperating users should be located as far away from each other as possible in order to limit the correlation of the experienced shadowing and improve the cooperation gain [5]. However, no more detailed model is available that captures at least two spatial domains, considers also the dependence on the frequency, or models the correlation in the case of multiple primary transmitters. For realistic modelling of DSA in larger scenarios with more than only few users a more complete representation of the spatial statistics of spectrum usage will be required. There is currently a lack of a theoretical framework capable of dealing with such models and allowing for rapid and reliable generation of realizations of such models. Measurement campaigns are also starting to generate vast amounts of data on spectrum behaviour, and there is need for efficient statistics allowing synthesis, sharing and comparison of these measurement results. Pure spectrum occupancy information from single point(s) is not sufficient due to, for example, the well-known hidden terminal problem and for DSA applications it is imperative to develop models capable to capturing spatial correlations of shadowing, spectrum occupancy etc. Also sharing simply raw data makes evaluation and use of measured spectrum information too complex and slow. Robust statistics and models are not only of theoretical importance, but also have practical impact as those can be used to plan spectrum utilization strategies and algoritms, and to dimension distributed spectrum sensing networks (see also the Radio Environment Map work in [12] for related discussion and applications).

In this paper we present selected techniques from spatial statistics and the theory of random fields that can be used as a foundation for creation of such more realistic spectrum models (see [13] for earlier application of random fields in propagation modeling). We cover some of the underlying theory both from analysis and modelling points of view, and apply these methods to data obtained from extensive spectrum measurements. The introduced techniques can reproduce several aspects of the spatial behaviour of spectrum and have great potential for analytical as well as simulation work. Artificial but realistic data sets can also be generated with low computational complexity.

The rest of the paper is structured as follows. In Section II we cover the basics of random fields and spatial statistics, focussing on metrics quantifying dependencies and correlations. In Section III we discuss the problem of creation of probabilistic models based on measured characteristics of the spectrum. These techniques are applied on measured data in Section IV, before drawing the conclusions and outlining future work in Section V.

II. Characterizing spectrum as a random field

Throughout the rest of the paper we adopt the probabilistic viewpoint towards analyzing and modelling spectrum. We treat measurement results, whether scalar-valued (such as mean power spectral density (PSD) on a given frequency band) or vectors, as a realization of some unknown *random field*. In general random fields can be thought of as extensions of the usual theory of stochastic processes from one-dimensional *time* to multi-dimensional *space*. In our case this space will be some domain D in either $\mathbb{R}^2$ or $\mathbb{R}^3$, depending on the setup of measurements and the models in question. Inclusion of various topological boundary conditions to account for finite size of the domain is trivially possible. As usual, the probabilities involved can either be interpreted as "true" randomness or simply as a model for the incompleteness of our knowledge on all the details of the system under study (Bayesian viewpoint). In this paper we shall deal mainly with purely spatial models (i.e. static models without time evolution) due to space limit. However, the generalization of the presented techniques to the spatio-temporal case is straightforward.

We shall begin by giving an overview of the different statistics that can be calculated for a random field Z [14] (see also the related discussion in [15] with applications in wireless sensor networks). Usual definitions of mean, variance etc. hold for Z unmodified, although now they yield functions on D instead of single numbers. For studying the dependencies of the values of Z at different locations we define the *covariance function* by

$$C(\mathbf{u},\mathbf{v}) \equiv \mathrm{Cov}\big(Z(\mathbf{u}), Z(\mathbf{v})\big), \tag{1}$$

that is, as the covariance of the values of Z at the locations $\mathbf{u},\mathbf{v} \in D$. This is the generalization of the usual autocovariance of time series analysis to the spatial case. Often instead of the covariance function the *semivariogram*

$$\gamma(\mathbf{u},\mathbf{v}) \equiv \frac{1}{2}\mathbb{E}\left\{|Z(\mathbf{u}) - Z(\mathbf{v})|^2\right\} \tag{2}$$

is used. It is directly related to the covariance function by

$$\gamma(\mathbf{u},\mathbf{v}) = \frac{1}{2}\big(C(\mathbf{u},\mathbf{u}) + C(\mathbf{v},\mathbf{v})\big) - C(\mathbf{u},\mathbf{v}). \tag{3}$$

We can often assume that Z is (second-order) stationary making γ and C functions of the separation $\mathbf{h} \equiv \mathbf{v} - \mathbf{u}$ only. The relationship between the semivariogram and the covariance function is then also simplified to $\gamma(\mathbf{h}) = C(\mathbf{0}) - C(\mathbf{h})$. If Z is further isotropic γ and C become functions of the distance $h \equiv \|\mathbf{h}\|$ only. Finally, we record the spatial analog of autocorrelation, namely the *correlogram* $\rho(\mathbf{h}) \equiv C(\mathbf{h})/C(\mathbf{0})$.

The selection between C, ρ and γ depends largely on personal tastes and particulars of the problem at hand. In a sense the semivariogram needs the weakest assumptions for its definition, and is relatively robust in the presence of spatial trends. There is also a well-defined theory for stochastic modelling and simulation based on semivariograms that can be applied in modelling of spatial phenomena (discussed in the next section). Before that, we discuss the problem of estimating second-order statistics of Z from observations Z_i made at n different locations $\mathbf{x}_i \in D$. Due to space reasons we focus on the estimation of the variogram, as well as on introducing two additional statistics that are particularly well suited for dealing with experimental spectrum data.

For estimating the variogram we usually employ a binning approach. Let $N(h)$ denote the set of pairs (i,j) such that $h - \Delta \le \|\mathbf{x}_j - \mathbf{x}_i\| \le h + \Delta$. Then the usual way of estimating the mean yields the *empirical semivariogram*

$$\hat{\gamma}(h) \equiv \frac{1}{2|N(h)|}\sum_{N(h)}(Z_j - Z_i)^2. \tag{4}$$

An index closely related to the correlogram is *Moran's I* defined in our notation by

$$I \equiv \frac{n\sum_{i,j} W_{ij}(Z_i - \bar{Z})(Z_j - \bar{Z})}{W\sum_i (Z_i - \bar{Z})^2}, \tag{5}$$

where W_{ij} is a matrix of *weights*, $\bar{Z}$ denotes the usual estimate for the mean of Z, and $W = \sum_{i,j} W_{ij}$. The value of Moran's I lies between -1 and $+1$, former indicating strong negative autocorrelation and latter strong positive autocorrelation. For independent Z_i we would have $I \approx 0$. Another commonly used metric for measuring spatial autocorrelation is Geary's C based on the squared differences in a manner analogous to the empirical semivariogram:

$$C = \frac{(n-1)\sum_{i,j} W_{ij}(Z_j - Z_i)^2}{2W\sum_i (Z_i - \bar{Z})^2}. \tag{6}$$

The value of C typically lies in the interval $[0, 2]$, with the value $C \approx 1$ indicating lack of spatial autocorrelation. Numerous variants of the I and C indices can be defined by choosing the weights W_{ij} appropriately. To obtain a single index for the whole domain D the W_{ij} are often made inversely proportional to the distance $\|\mathbf{x}_j - \mathbf{x}_i\|$. Distance-dependent indices can be obtained by giving positive weight to pairs of locations in the radial binning fashion used to estimate the variogram above.

We conclude this section by briefly mentioning alternatives and extensions to the correlation metrics introduced above. Integral transformations, such as Fourier or wavelet transformations and multipole expansions of the random field are commonly used [16]. These form the obvious extension of Fourier/wavelet analysis to the spatial domain. A natural approach in stochastic geometry would be to treat measurement locations as a realization of a *point process*, with measurement values being interpreted as marks on the points. The second-order structure could then be recovered by studying mark correlations [17]. Measurements performed on a regular lattice can also be thresholded and interpreted as random images. See [18] for an introduction of the related analysis methods. Finally, techniques for characterizing self-similarity, such as the Hurst exponent and fractal dimension can be applied to random fields as well. See [19] for example theory and relevant models.

III. STOCHASTIC MODELLING OF SPECTRUM

In the previous section we discussed metrics suitable for characterizing (second-order) stochastic spatial structure of spectrum. We shall now focus on the problem of *spectrum modelling*, that is, design of stochastic models with similar second-order structure to measured spectrum. In this context a spectrum model is required to capture the dependency of the PSD on time, frequency and space. Additionally, antenna aspects such as polarization and angle of arrival could be added for improved realism. The applications of such models are obviously manifold. First, they can be used as parts of simulation environments as well as analytical calculations when studying different aspects of cognitive radio operation and DSA. Secondly, such models could be incorporated into the reasoning and modelling processes of cognitive radios themselves. The probabilistic and Bayesian nature of typical random field models is well suited for this task. We have elsewhere described our approach to develop a *topology engine* for cognitive radios as a part of our cognitive resource manager (CRM) framework which could be used as a foundation for these processes [20], [21]. Finally, these techniques could be used by operators and regulators for what-if-analysis and stochastic interference characterization in network planning.

We shall consider two distinct modelling approaches. First, we study techniques that attempt to model spectrum directly from a set of measurements. Second, we consider a more "first-principles" approach, in which we model the locations and transmission characteristics of the transceivers, and obtain the spectrum model as a derived quantity.

Usually the fitting of a random field model to experimental data is performed in two phases. First, based on estimates of the covariance function or the semivariogram an analytical model of the desired covariance metric is fitted. Then, based on the obtained fit, a model is chosen and further parameters are tuned to comply with the fitted covariance model. More direct approaches become possible with very simple models (such as spatial generalizations of AR and MA models for time series or Markov models, see [22] and references therein). However, this comes at a cost of losing control over the second-order structure. Since correlations in spectrum seem fundamental to many cognitive radio applications, we shall focus on methods explicitly taking them into account. In particular we focus on methods based on semivariograms. There also exists a number of tools implementing the techniques presented. These include, for example, the `spatial` [23] or `RandomFields` [24] packages in the R environment [25].

A. Detailed modelling process

As discussed before, the first step is fitting the semivariogram model. The empirical variogram defined above does not suit this purpose directly as it might not have the asymptotic properties a proper semivariogram has. Instead it can be used to guide the selection of the appropriate semivariogram model. Common choices are the *exponential model*

$$\gamma_{\text{exp}}(h) = a + b\big(1 - \exp(-h/c)\big), \tag{7}$$

with $a, b, c \in \mathbb{R}^+$, the *Gaussian model*

$$\gamma_{\text{G}}(h) = a + b\big[1 - \exp\big(-(h/c)^2\big)\big], \tag{8}$$

and the very general *Matérn model*

$$\gamma_{\text{M}}(h) = a + b\left(1 - \frac{1}{2^{\alpha-1}\Gamma(\alpha)}\left(\frac{h}{c}\right)^{\alpha} K_\alpha\left(\frac{h}{c}\right)\right), \tag{9}$$

where Γ is the gamma-function and K_α is the modified Bessel function of the second kind. The Matérn model gives the exponential and Gaussian models as special cases $\alpha = 1/2$ and $\alpha \to \infty$, respectively. The selection of the parameters of the model can be performed by least-squares fitting or, preferably, maximum likelihood estimation.

After the semivariogram model is selected and fitted we can proceed with model selection for the random field. If only a single realization is required we can obtain one by *kriging* [14], that is, finding the best linear unbiased estimation (also known as Wiener-Kolmogorov estimate) of the random field values between the measurement points. Kriging can thus be thought of as a kind of stochastic version of linear interpolation. The equation defining the model is simply

$$\hat{Z}(\mathbf{x}) = \sum_{i=1}^{n} w_i Z_i, \tag{10}$$

where the weights w_i are chosen to minimize the prediction error. For details of the related optimization problem and its solution we refer the reader to [14]. In order to obtain multiple realizations we need to settle for a concrete model for the random field. Since we are primarily interested in modelling correlations in spectrum properties, a particularly appropriate choice are *Gaussian random fields*. These are define by requiring that all joint distributions of the values $Z(\mathbf{y}_i)$ at m different locations $\mathbf{y}_i$ are m-variate Gaussians. Since these Gaussians are completely defined in terms of their means and covariance matrices, knowledge of the covariance function or the variogram together with the empirical mean uniquely defines the model. Realizations of such a Gaussian random field can then be generated by Monte Carlo techniques either directly or combined with Fourier transforms (see, for example, [26]). We shall give an example of the described workflow from variogram estimation to kriging in the following section, using measured spectrum data as the basis.

B. Modelling the transmitter distribution

Before applying the above-described machinery on experimental data we briefly outline an alternative approach for spectrum modelling, based on the use of stochastic models of transmitter locations. In earlier work we have studied modelling and characterization of node locations in wireless networks using the stochastic geometry of point processes [27], [28]. In particular, we have shown that it is possible to create probabilistic models on the location distributions that faithfully represent the second-order statistics of locations and densities of the nodes. Such a model can be used to define a random field by assigning each node a transmission model (with possible complexity ranging from continuous omnidirectional transmission at constant power to time-dependent transmission powers and directionality) and prescribing a model for propagation. The resulting random field is then of the form

$$Z(\mathbf{y}) = \sum_{\mathbf{x}_i \in N} P(\mathbf{x}_i, \mathbf{y}) T_i(\theta_{\mathbf{x}_i \to \mathbf{y}}), \qquad (11)$$

where $N = \{\mathbf{x}_1, \ldots, \mathbf{x}_n\}$ is the point process model of node locations, P is the model for path loss (either deterministic or stochastic), $T_i(\theta)$ is the transmission power of node i into direction θ and $\theta_{\mathbf{x}_i \to \mathbf{y}}$ is the direction from $\mathbf{x}_i$ to $\mathbf{y}$. All of these factors can of course be made time-dependent yielding a spatio-temporal spectral model.

IV. Examples from spectrum measurements

After introducing models for different aspects of spatial statistics we continue with giving few examples for these metrics based on distributed spectrum occupancy measurements.

A. Measurement setup

The deployed measurement system is an extension of our previous stationary setup described in [29]. Since accurately measuring the spectrum at a large number of locations in parallel is difficult in practise we measured the spectrum at multiple locations of a regular grid consecutively. At parallel we kept a second identical setup at a single location. The results gathered with the latter one proved that the spectrum usage throughout the measurement time can be assumed to be wide sense stationary for several services. Both setups were time-synchronized using GPS-receivers and are based on Rohde & Schwarz FSL6 portable and battery-powered spectrum analyzers. We used omnidirectional antennas as described in [29] and evaluated frequency bands used by the most popular wireless services. The used resolution bandwidth was 100 kHz and the time between consecutive sweeps was 1 sec.

Most of the study was performed in the downtown area of Aachen, Germany, using two different grid sizes, in the following called *AC*. We investigated shorter range systems such as DECT or WLAN using a grid size of 15 m and mid-range technologies, such as DVB-T, GSM, UMTS, etc. using a grid size of 250 m. Additionally, we had the opportunity to carry out another campaign during the CeBIT industry fair in Hannover, Germany, in the following called *CeBIT*. Both campaigns were done in March 2008.

B. Geary's and Moran's I

We computed Geary's C and Moran's I for all available data sets after averaging all samples taken at each location. The involved weighting factors W_{ij} were chosen as $\frac{1}{d_{ij}}$, where d denotes the distance between the compared measurement locations $\mathbf{x}_i$ and $\mathbf{x}_j$. In the following we present the behaviour of these metrics over frequency as well as over distance. In the latter case a distance binning was applied as described in section II.

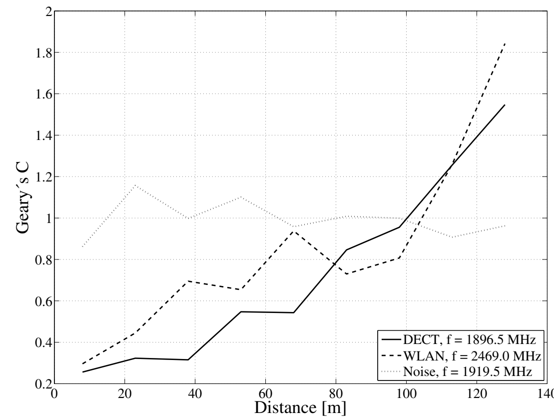

Fig. 1. Geary's C extracted from our *AC* measurement results for selected frequencies in the DECT and WLAN bands.

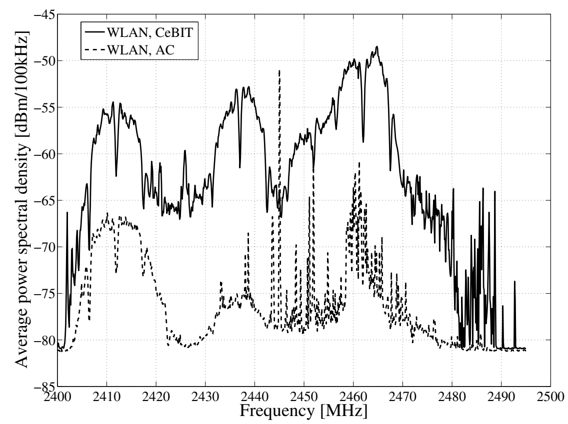

Fig. 2. Average PSD measured in the ISM-band at 2.4 GHz outdoors in *AC* and indoors at *CeBIT*.

Figure 1 shows Geary's C over the distance for selected *AC* results. The curve for the noise signalfluctuates around the value of one as expected for noise-like signals. The two other shown curves for WLAN and DECT services increase since the similarity of the measurements decreases over distance. For such short range services with strictly limited transmit powers we expect such behaviour. This case validates that the presented metric is able to capture spatial statistics of spectrum.

Figure 2 shows the PSD as measured in the 2.4 GHz ISM-band averaged over time. At both locations WLAN was by far the most often used service in the investigated spectrum band. For the *CeBIT* measurements the most popular channels 1, 6, and 11 can clearly be identified. In the *AC* case this structure is less clear because the overall amount of detected WLAN-traffic was lower. Additionally, narrowband interference from microwave ovens around the frequency of 2450 MHz can be identified.

Figure 3 shows Geary's C over frequency calculated from

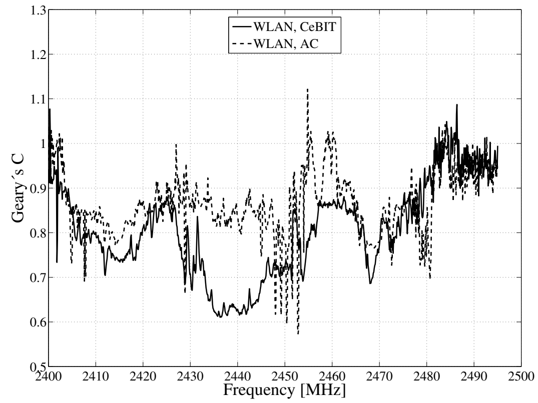

Fig. 3. Geary's C computed from the measurement results gathered in the 2.4 GHz ISM band outdoors in *AC* and indoors at *CeBIT*.

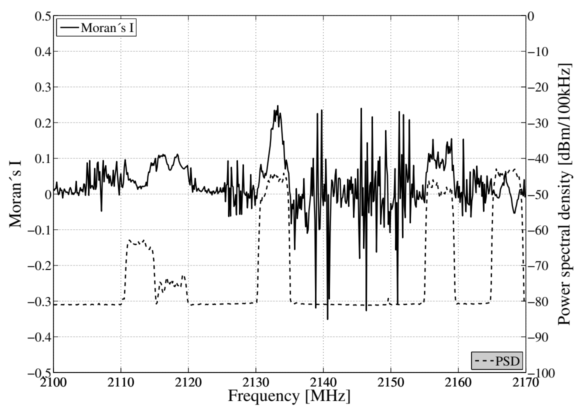

Fig. 4. Moran's I computed from the results measured in the UMTS downlink band in *AC* using the larger grid size. The corresponding received PSD is also shown.

the same data sets. For the *CeBIT* case the channel structure can also be identified in this chart since the WLAN usage was very high at all measurement locations. Again, the values fluctuate around one for noise-like signals as were found at frequencies above the upper limit of the ISM-band at 2483.5 MHz. The microwave interference in *AC* was rather strong and seems to be well distributed throughout the whole covered area because the computed C-values are the lowest ones throughout the whole band indicating high similarity. Additionally, the basic WLAN-channel structure can also be partially identified for the *AC* results. These results for a less busy location also show that sufficient amount of data is required since existing similarity patterns might be missed if the investigated bands are only rarely used as might be the case, e.g., for cellular uplink bands.

Figure 4 shows an example for Moran's I. We evaluated the data measured within the UMTS downlink bank in *AC* using the larger grid size. Since UMTS is based on CDMA technology base stations transmit continuous signals and announce their presence using at least the broadcast code to new client devices trying to join the network. Additionally, no feedback is available for such common channels and significantly less transmit power adaptations are conducted by the base stations. Resulting from these facts the measured PSD, which is also given in Figure 4, shows noise-like behaviour over time although on a significantly higher power level. Since Moran's I applies normalization to the averaged PSD levels this difference in PSD does not have considerable impact and Moran's I is similar for used and unused UMTS channels. However, in used channels the measured PSD fluctuates less over space because all man-made interference is masked by the much stronger UMTS downlink signal. Hence, the changes of Moran's I between frequencies belonging to the same UMTS channel are smaller compared to unused channels.

Although Moran's I is very helpful in other scenarios, that we leave out here due to space limitations, the example also shows the limitations of the presented techniques. They are very helpful tools in the modelling and evaluation of spatial statistics but have to be applied carefully bearing in mind the characteristics of the signals and systems under test.

C. Semivariogram and Kriged estimate

In the following we will also give a short example on further metrics as introduced in section II. We describe the modelling process for the average PSD and again use the UMTS downlink data gathered in *AC*. The covered area includes multiple base stations and our artificially generated model also shows this characteristic.

We begin the modelling process by choosing a model for the semivariogram and performing the parameter fitting. The empirical semivariogram can be calculated directly from (4). The shape of the resulting estimate indicated that the Matérn model (9) should provide a good fit, so the Matérn model was chosen for further analysis. Figure 5 depicts the empirical semivariogram together with the Matérn model obtained by least squares fitting. Based on the obtained model we then created a realization of the corresponding mean PSD model by kriging. The outcome is shown in Figure 6, the colour intensity indicating the mean PSD in the realization of the model. The yellow highlights correspond approximately to the locations where the UMTS based stations would be deployed in such a artificially generated scenario.

V. Conclusions

In this paper we argued that more accurate models for spectrum usage are required in multiple areas of cognitive radio research. Compact and concise mathematical formulations that realistically describe the spectrum usage would greatly improve the current theoretical work. Artificially generated data from such models would also enhance the simulation-based research in terms of realism and comparability. Also in the practical domain, measurement groups would benefit if they could characterize results with few well known parameters instead of exchanging large amount of digital data or relying only on simplified spectrum occupancy statistics. Finally, it is to be expected that also cognitive radio devices

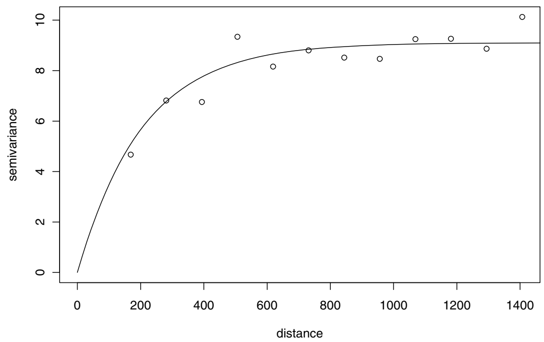

Fig. 5. The empirical semivariogram (circles) and fitted Matérn semivariogram model for the mean PSD of a selected frequency in a used UMTS downlink channel in *AC*.

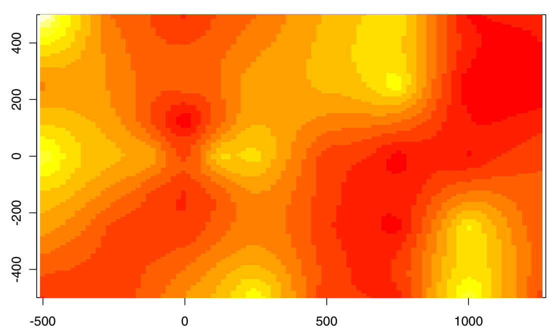

Fig. 6. Kriged estimate for the mean PSD of a selected frequency in a used UMTS downlink channel in *AC*.

themselves would benefit if those could use simplified models to infer the radio environment they are operating in. The presented methodology has also applications beyond cognitive radios systems and we are considering to use it as an extra tool to build interference optimized WLAN channel allocation strategies [30].

We discussed spatial statistics and the theory of random fields as promising candidates for such models. Based on our measurements, we also presented results that show the potential of the methods. As part of our future work we will extend the presented evaluation of our measurement data and investigate how cognitive radios could build up and exploit such information during operation.

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