Influence of Node Location Distributions on the Structure of Ad Hoc and Mesh Networks

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***Abstract*—We study the impact of the distribution of node locations on key properties of wireless networks. In particular, using Monte Carlo simulations we study in detail the behavior of average path length for different distributions and parameters thereof. We also show that distribution of client locations has significant impact on network planning, in particular on combination of base station placement and range assignment. In both cases our results indicate that the underlying point distributions of nodes have clear effects on the wireless network properties. We also discuss the implications of our results and the role of Poisson point process as an approximation of other distributions.**

I. Introduction

The study of unstructured and self-organized networks has become very popular during the last decade. The research of these networks is going beyond the earlier interest on ad hoc networks, as also different mesh, relay and sensor network concepts have been recently introduced as potentially distributive new technologies. One of the key points when studying these networks, both theoretically and practically, is to understand the fundamental opportunities and limitations with regards to connectivity, capacity, throughput and latency. Seminal works in this domain include Gupta and Kumar [1] who derived the capacity bounds for two-dimensional wireless networks, Xue and Kumar; and Gupta and Kumar [2], [3] on critical transmission power to provide full connectivity in the network. A lot of subsequent work has been done especially to understand the connectivity and capacity-delay bounds for such networks in [4].

In all of the works mentioned so far, the common underlying assumption has been that the nodes in the network are uniformly randomly distributed. However, this assumption does not hold for real networks and could be considered only as an approximation for conducting simplified studies. Only recently some groups have started to consider more closely the effects of the wireless network topology, more specifically the underlying point distribution of nodes, to different metrics. The studies by Foh et al. [5], Li et al. [6], Santi et al. [7] and Petrova et al. [8] are also taking into account non-uniform node distributions in the connectivity analysis. The initial results have shown that the underlying point process that generates node locations strongly affects the network connectivity. It has been also shown that the spatial distribution of nodes can be highly correlated in wide scale of distances, see e.g. a recent study showing spatial correlations for Wi-Fi access points in the USA [9].

This paper is an extension of our earlier connectivity analysis [8]. It makes two key contributions. First, we apply known point distribution processes, both clustered and non-clustered, e.g., *Poisson point process*, *Thomas process*, *Matérn hard-core process* and the so called *Strauss process* [10] to test the effects of topology for self-organizing wireless networks. In contrast to the earlier works, we are also interested in to know how much the underlying distribution affects the average path length of the network, while at the same time studying the connectivity of the network. We also study the impact the client node distribution has on network planning, namely placement and range assignment of base stations or access points. We formulate the corresponding optimization problem as a variation of the well-known *plate placement problem* of geometry, and use genetic algorithms (GAs) to obtain high-quality solutions. We specifically show that the distribution of client nodes has a major impact on the number of base stations required, as well as the distribution of coverage radii. These results have clear practical consequences as they highlight the strong influence client distributions have on dimensioning problems in wireless networks.

The rest of the paper is organized as follows. In Section II we describe in detail the models used in the rest of the paper. In Section III we study the behavior of average path length for the different processes using Monte Carlo simulations. The influence of node locations on network planning problems is then discussed in Section IV. Finally, we draw our conclusions, discuss the implications of the results and suggest items for future work in Section V.

II. Models used

We shall now give a more detailed description of the point processes used as models for node locations in this paper. As a baseline we shall use the (homogeneous) *Poisson point process* (PPP). We create realizations of PPP by first selecting the average number of nodes λ per unit area and drawing a sample n from Poisson distribution with parameter λ. We then assign the x and y coordinates of n nodes to be uniformly distributed on $[0, 1]$. The result is a realisation of PPP on the unit square.

The next model we shall consider is the *Thomas process*, which is an example of a *Poisson cluster process*. These are in general constructed by using a PPP as a distribution of cluster centers, and then generating for each cluster center a collection of cluster points. In the case of the Thomas process (TPP) the number of points in each cluster has Poisson distribution with parameter μ (giving the average number of points in each cluster), and the locations of the cluster points follow two-dimensional normal distribution centered on the cluster center. Only the cluster points are retained in the final point pattern, yielding on average $\lambda\mu$ points on our unit square. Usually the normal distribution of cluster point locations is assumed to be symmetric with variance σ^2, but we also consider the case with covariance matrix of the form $\mathrm{diag}(\sigma_x, \sigma_y)$.

The third process we consider is an example of a *hard-core process* in which points are not allowed to lie closer to each other than a prescribed minimum separation, called the *hard-core radius*. The particular construction we shall use is the *Matérn hard-core process* [10]. We again start with a Poisson point process. Then for each point we randomly assign *fitness* from the interval $[0, 1]$. To complete the construction, whenever two points lie closer than the hard-core distance apart, the node with the highest fitness is retained and others deleted.

The fourth and final model considered here allows to smoothly interpolate between the PPP and hard-core model. We consider fixed number n of points $\{X_i\}$, with probability density of the locations given by

$$f(X_1, \ldots, X_n) = \exp\bigl(-H(X_1, \ldots, X_n)\bigr)/Z, \tag{1}$$

where Z is chosen to ensure normalization by

$$Z = \int_{\mathbb{R}^{2n}} \exp\bigl(-H(X_1, \ldots, X_n)\bigr)\, dX_1 \cdots dX_n. \tag{2}$$

Such a point process defined in terms of the density f is called a *Gibbs process*. The Gibbs model is, of course, extremely general. It becomes much more useful by assuming the *pairwise interaction model* defined by

$$H(X_1, \ldots, X_n) = a_0 + \sum_{i=1}^{n} \psi(X_i) + \sum_{1\le i<j\le n} \phi(X_i, X_j) \tag{3}$$

where $\psi : \mathbb{R}^2 \to \mathbb{R}$ and $\phi : \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$ define the so-called single-particle and pair contributions to H. Assuming that the overall density of the points is homogeneous gives the restrictions $\psi(X_i) \equiv a_1 \in \mathbb{R}$ and $\phi(X_i, X_j) \equiv \theta(\|X_j - X_i\|)$, where $\theta : [0, \infty) \to (-\infty, \infty]$ is the *pair potential*. Common choices of the pair potential include the *hard-core model*

$$\theta_{h,b,R}(r) = \begin{cases} \infty & r \le h \\ -b & h < r \le R \\ 0 & r > R \end{cases} \tag{4}$$

and the *Lennard-Jones potential*

$$\theta_{\alpha,\beta,\sigma,n,m}(r) = \beta \left(\frac{\sigma}{r}\right)^n - \alpha \left(\frac{\sigma}{r}\right)^m, \tag{5}$$

where $\alpha \ge 0$, $\sigma > 0$ and $n > m$. The special case of $h = 0$ and $b < 0$ of the hard-core model is called the *Strauss process*,

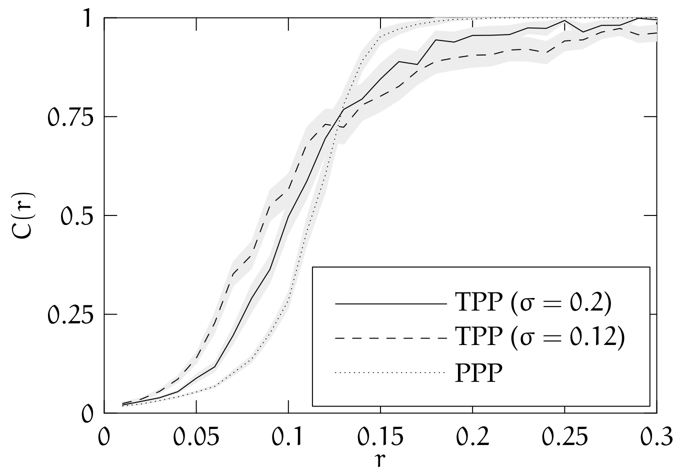

Fig. 1. Connectivity of a random geometric graph with different underlying point distributions.

which we use in the following. The probability density of the Strauss process can also be written in the form

$$f(X_1, \ldots, X_n) = \alpha\beta^n\gamma^{s(X_1,\ldots,X_n)}, \tag{6}$$

where α is a normalization factor, β controls the intensity of the process, s is a function counting the number of pairs closer than distance R apart, and $0 \le \gamma \le 1$ is an interaction parameter.

To a point process N we now associate a graph $G(N, r)$ obtained by letting N give the vertices of the graphs, and connecting those vertices by edges that are closer than distance r apart. Such a graph is called the *random geometric graph* of N, and is perhaps the simplest non-trivial model of a wireless network. In [8] we studied *connectivity* of such graphs, defined as the fraction $C(r)$ of the nodes connected to the largest component of $G(N, r)$. Figure 1 shows an example of the impact the distribution of node locations has on $C(r)$. In this paper we extend this study towards another network property of interest, namely the *average path length* $\ell(G)$. For this study we define $\ell(G)$ as the average of the length of the shortest path between vertices v and w over all *connected* pairs of vertices.

III. Simulation results

In this section we present the simulation results for the behavior of the average path length $\ell(G)$ of a random geometric graph $G(N, R)$ for different choices of N and R. For generating random geometric graphs all models were adjusted to have the same mean intensity, namely 100 points per unit area. Such a limited intensity was chosen to yield results applicable to networks of limited size. Thomas process was realized with on the average five clusters per unit area. This yields the average of twenty points per cluster to ensure the desired overall intensity. The cluster size parameters used were $\sigma = 0.20$ for large clusters and $\sigma = 0.08$ for small clusters. For the Matérn process the core radius was set to be 0.03. The interaction parameter γ for the Strauss process was set

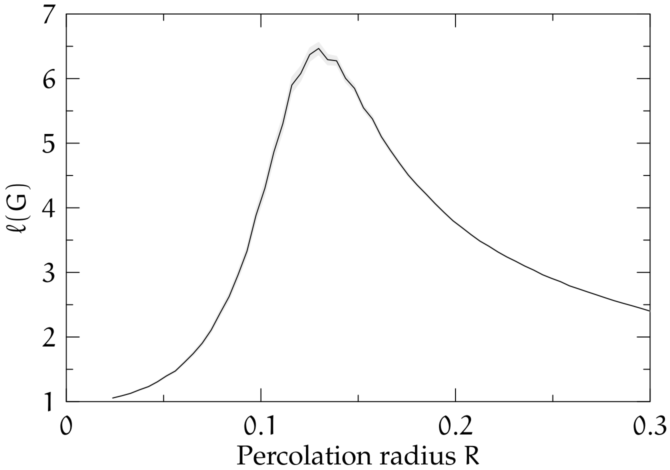

Fig. 2. Average path length for the case of the Poisson point process.

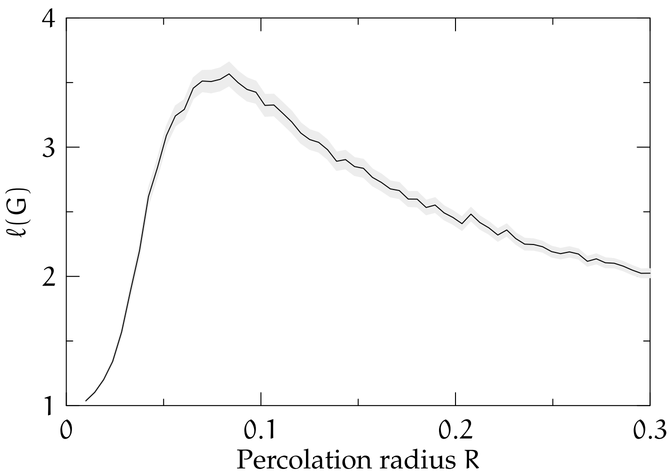

Fig. 3. Average path length for the Thomas process with $\sigma = 0.08$.

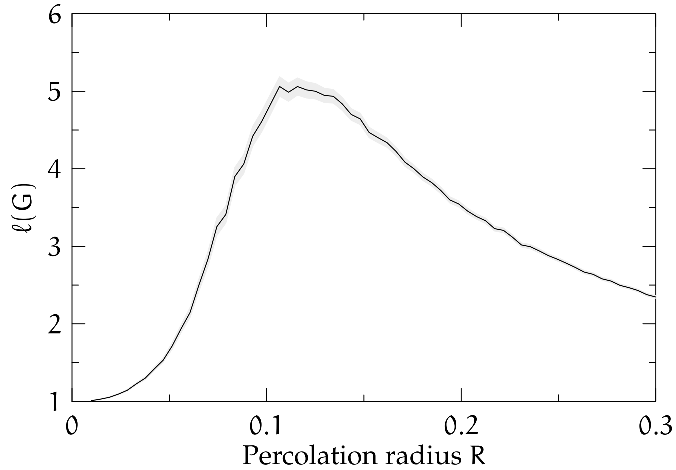

Fig. 4. Average path length for the Thomas process with $\sigma = 0.20$.

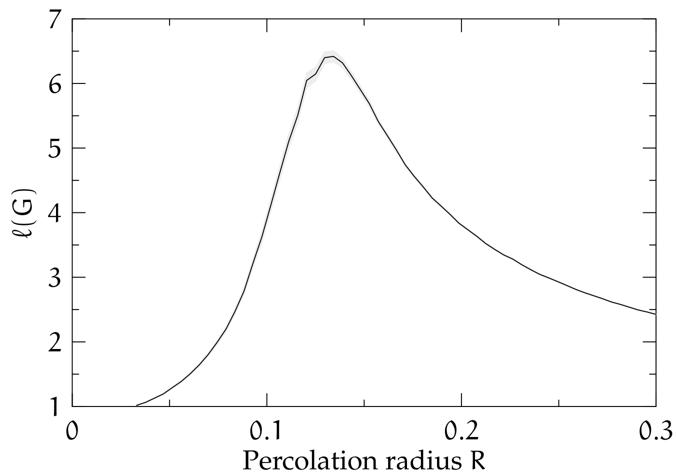

Fig. 5. Average path length for the case of the Matérn hard-core process.

to 0.3. In each case the mean value of the said quantity over 1000 simulation realization with the given parameter set will be given, together with the 95% confidence interval for the standard error.

Figure 2 shows the behavior of the average path length for the PPP case. The high peak slightly past $R = 0.1$ corresponds to the case of $G(N, R)$ becoming almost fully connected. We see that the average path length drops relatively rapidly afterwards as R is increased. This observation is of importance especially to topology control. Obviously a careful tuning of transmit powers to yield a barely connected network at the same time maximizes average path length. However, the results clearly show that by relatively slight increase of the range of connectivity, average path length can be cut to half. This should be contrasted with Figures 3 and 4 showing ℓ for the Thomas process case. Due to the clustered nature of the distribution the path lengths are in general lower, and the decrease as R is increased is much more gradual.

For processes of hard-core type the behavior of $\ell(G)$ is much closer to the Poisson case. In the case of the Matérn process, Figure 5 shows almost exactly the same behavior as seen for the PPP case in Figure 2. The results for the Strauss process, seen in Figure 6, are quite similar to the PPP and Matérn cases, although there is a notable quantitative difference.

We have seen that the average path length of a geometric graph is relatively sensitive to the underlying point distribution. Especially clustered node locations lead to significantly different results than for hard-core or Poisson distributions. Since $\ell(G)$ is a network property with major impact on performance, these results strongly support the use of diverse node location distributions when doing simulation studies on wireless networks. Just relying on the customary Poisson approach can lead to results of limited applicability in more clustered networks. These lessons are also important to remember in various network deployment problems as well as when reasoning about power control.

IV. Influence on Network Planning

The network planning problem we are addressing in this section is placement of base stations (BS) and/or access points

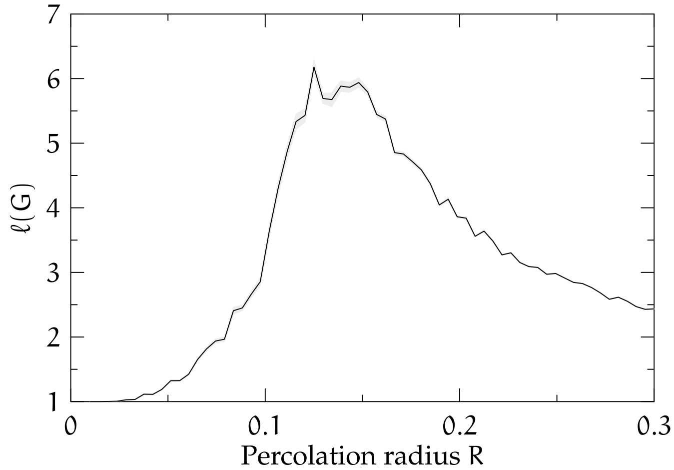

Fig. 6. Average path length for the case of the Strauss process.

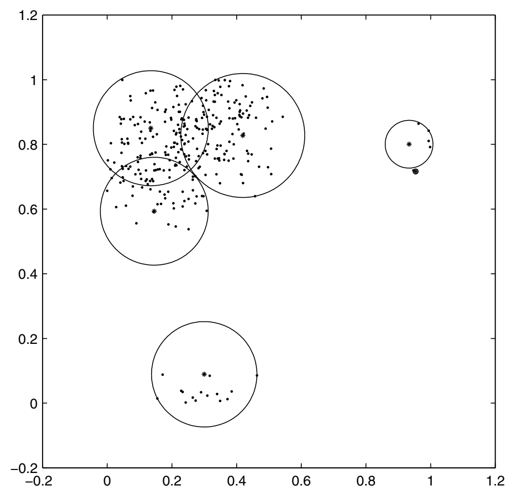

Fig. 7. Result of a combined base station placement and range assignment for Thomas process with $\sigma = 0.1$.

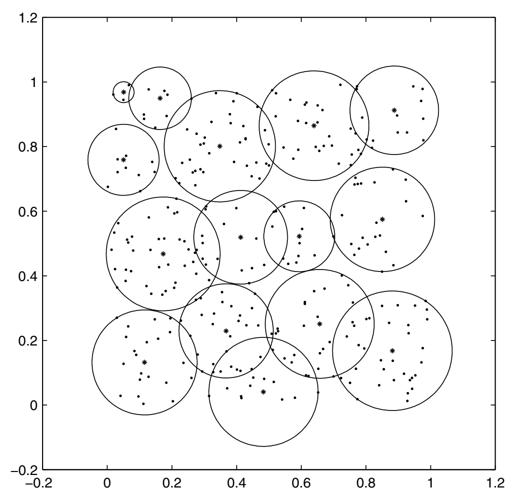

Fig. 8. Result of a combined base station placement and range assignment for Thomas process with $\sigma = 0.5$. The results for Poisson node locations are in practise similar.

(AP) into a field so that the interference is minimized, while at the same time a minimal number of BS is ensured and a full connectivity is achieved by minimal number of hops between the clients and the BSs. This problem is also known as *plate placement problem* (for some of the related problems in network planning see, for example, [11], [12]).

The general question in the plate placement problem is how many plates (various sizes also allowed) are needed to cover a given area so that the number of those is minimized and the overlap is minimized (or no overlap is allowed, and the non covered area is minimized). This is, of course, known problem and the methodology has been applied e.g. to 3D-frequency planning in the cellular networks [13]. In our particular case we allow for BSs to overlap in the coverage which will be penalized by increased interference in the overlapping regions.

The main goal is to study the effect of the underlying point distribution of the clients has on the BS placement. What in fact have to be solved is a multi-parameter optimization problem where as a result we get the position of the BSs so that the interference and the number of BSs is minimized and the connection radius r (transmission power) of the BSs is optimal given number of client distributions. Due to fact that the interference calculation makes the system model non-linear it is beyond the scope of this paper to try to find any analytical solution. We use genetic algorithm (GA) to find the optimal solution [14], [15]. Genetic algorithms have a unique niche in the area of multi-objective optimization. Due to the parallel search nature of the algorithms, they are able to explore the search space in several locations at the same time and find the optimal solution relatively fast. The use of GA itself for this problem is nothing particularly interesting so we do not bring any further details regarding its operation in this paper.

We studied the number of BSs and the resulting range assignments for a number of point processes. To estimate the interference of the resulting point configuration we apply simplified propagation model with Rayleigh fading and path loss exponent of $\alpha = 4$. Figures 7 and 8 give representative examples on solutions for the BS placement problem. Both figures depict solutions in the case of the underlying client distribution being generated from the Thomas process with $\lambda = 2$ and $\mu = 150$ but with different cluster spreads σ. We see that the influence of varying σ has a major impact both on the required number of base stations as well as on the distributions of ranges assigned. In the high-spread case with $\sigma = 0.5$ (which approximates well the results for the Poisson process as well) a larger number of BSs is needed, with more homogeneous ranges. This clearly shows that the underlying point distributions have a major effect on network planning problems.

V. Conclusions

We have demonstrated that the underlying point distribution of nodes have clear effects on the wireless network properties. Although this is, in a sense, quite straightforward the implications have not been stressed enough qualitatively or studied quantitatively. Our work is giving specific quantitative results in Section III and qualitative discussion in Section IV. It is not possible to state that PPP results provide, for example, consistently the worst case results. The situation is rather subtle, and in this article we have shown that for example the partial connectivity (percolation) ratio is depended on the point distribution. We have shown that, if one were comparing PPP against TPP, the partial connectivity ratio is higher for TPP than for PPP in the case of low value of connectivity radius, r. This situation is, however, opposite for larger values of r. This shows that one can not make any overly simplified deductions from simple PPP based modeling.

The indicative results shown in Section III and IV also show that topology information can have direct and strong implications for network design and selection of appropriate protocols for self-organizing networks. As an example we note that for the highly clustered models, such as TPP with $\sigma = 0.12$, very high connectivity radius values are required before the full connectivity is reached. In fact, the value might be prohibitively high, since due to interference and packet collisions the throughput may be lowered too significantly. The differences in average path length statistics also demonstrate clearly that *if* the system knew in which kind of topological environment it is operating in it could choose to use, say, different routing algorithms to better exploit the underlying topology. This has lead us to conclude that the future smart networks should be topology aware so that they could autonomously select the best operating parameters and protocols. One should also note that the same logic can be used in reverse fashion. If it is possible to decide on the topology of the deployed network, e.g. in the case of sensor networks, the network designer or some automated topology control mechanism might want to chose topologies that fit with some particular needs or fixed protocols. As far as we are aware of, this possibility has not been explicitly pointed out or studied in previous works.

Apart from the obvious applications with the ad hoc, mesh and sensor network we foresee applicability with the emerging cognitive wireless networks. In [16] we have suggested an entity called Topology Engine for cognitive radios, which could exploit the above mentioned topology information to make smart decisions. This approach has also clear relevance in the context of Radio Environment Maps (REM) approach proposed by VirginiaTech [17]. We also expect that topology affects to various spectrum allocation schemes, e.g. topology information may be useful for WLAN frequency allocation [18].

The main contribution of this paper has been to demonstrate also quantitatively that the underlying node distribution has indeed noticeable effects to many metrics of practical networks. The particular results themselves should not be over interpreted, since our main purpose has been to show that it is highly important to start to consider more complex and realistic point distributions than PPP in analysis of wireless networks. We would also like to stress that nevertheless also for many analysis purposes using PPP is still justified. Many results can be analytically derived for PPP, but considering more complex distribution make some problems analytically intractable and one has to consider only Monte Carlo simulations. Thus the PPP analysis still provides interesting boundary conditions, it is useful as test or reference cases, and in certain limits can be even realistic.

Acknowledgment

The authors would like to thank RWTH Aachen University and the Deutsche Forschungsgemeinschaft (DFG) for providing financial support through the UMIC excellence cluster as well as the European Commission for the financial support through the project ARAGORN. We also warmly thank Maria Michalopoulu for helping us with the optimization software for Section IV and producing a part of the figures.

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