Effects of Topology on Local Throughput-Capacity of Ad Hoc Networks

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***Abstract*—Most publications on the capacity and performance of wireless ad hoc networks share the underlying assumption of a uniform random distribution of nodes. In this paper, we study the effects of different node distributions on the local throughput of the slotted ALOHA MAC protocol. The throughput achieved in a network where the nodes are distributed according to a Poisson point process is used as a baseline performance measure for the comparison with other point distributions. Our simulations show that non–uniform random node distributions have a strong impact on the local throughput which is related to the network capacity and performance. This means that the node topology should be taken into account in more detailed analyses and simulations of ad hoc, sensor, and mesh networks.**

I. Introduction

There has been an increasing interest in the study and the deployment of different unstructured networks such as ad hoc, mesh, and sensor networks. These networks have a high potential to enable new applications and they promise to achieve cost–savings, self–organization, and enhanced reliability. In principle, the model for these networks is a group of communicating nodes, which build wireless communication links between each other without centralized control. In the case of such networks, there are two fundamental aspects to understand and model, namely the connectivity and the network capacity.

A great deal of work has been done to understand the connectivity regions (percolation) of these networks. Recently, a number of fundamental papers have been published to increase our understanding of the capacity of networks. For example, Gupta and Kumar [1] derived capacity bounds for wireless (ad hoc) networks in their seminal paper. Furthermore, Gupta and Kumar [2] and Xue and Kumar [3] derived results for the critical transmission power needed to build a fully connected network. Bettstetter [4] has studied the number of nodes needed to maintain k–connectivity. El Gamal *et al.* [5] have recently investigated the optimal throughput–delay scaling in wireless networks in great detail.

Most of the previous works are based on the assumption that the network nodes are uniformly randomly distributed over the studied area. However, it is not clear that such an assumption holds for real wireless networks. For example, the authors of [6] analyzed the WLAN access point distribution in the USA and showed that their distribution is highly correlated. Similar studies have been done on the distributions of user equipments. Although the non–random distribution may be intuitively clear, it has not been emphasized and studied rigorously enough.

Recently, some of us have discussed the connectivity analysis of non–uniformly random clustered ad hoc networks in [7]. The paper showed that the underlying node distribution can have not only quantitative but also qualitative effects on the network. Other exceptions considering different clustered models include a work by Santi [8], in which the Random Waypoint Model was used to generate non–uniform point distributions for the connectivity analysis. Ferrari and Tonguz studied in [9] the effects of clustering on the bit error rate (BER) performance of ad hoc networks. Therefore, they introduced a so–called 'uniformly clustered' distribution, where cluster parent points are aligned on a square grid and daughter nodes of each cluster are uniformly distributed on a disc centered at the parent point. A recent but yet unpublished work is [10]. The authors derived analytical expressions for the interference and transmission success probability in clustered ad hoc networks for different fading environments. Finally, Foh *et al.* [11] and Li *et al.* [12] have studied connectivity with non–uniform distributions in the 1–dimensional case.

In this paper, we extend our previous connectivity analysis presented in [7] to show specifically that also the local throughput of wireless networks is strongly affected by the underlying point distribution of the nodes. Although these results provide us with information on the node capacity and there is an obvious relation to the network capacity, we refer all the results in the following as local throughput statistics.

The rest of the paper is organized as follows: In Section II, we give an overview of the communication model used for the throughput analysis. Section III characterizes and justifies the different point processes we use to study their influence on the network capacity. In Section IV, we show simulation results. Section V concludes the paper.

II. Communication Model for Capacity Analysis

We assume a simplified model where each node is operating the slotted ALOHA MAC protocol [13], has the same packet transmission probability p during each time–slot, and an equal transmission range R. Although the slotted ALOHA protocol is simple, it is well suited for our purposes. First, the ALOHA protocol is not without practical value as it is used as a bootstrapping protocol in many systems and it is also employed in 2G and 3G cellular systems for particular purposes. The main benefit is that the ALOHA protocol and numerous variations thereof have been widely researched both theoretically and experimentally and, therefore, a large amount of results is readily available for comparison. Moreover, the theoretical results can be used to validate our simulation results in the case of the Poisson point process (PPP) (unified random distribution of nodes).

In the performance analysis, normally one of the two following scenarios is considered. In the *non–capture case*, a transmission is successful if only one node within a certain distance from the destination transmits and the destination itself is silent. One therefore defines a connection range within the nodes can communicate. Nodes outside this range have no interference impact and are ignored. In the *capture case*, a transmission is only successful if the signal–to–interference–ratio (SIR) is above a certain, technology–defined detection threshold and the destination does not transmit. Hence, transmitter–receiver distances, transmission powers, and the channel attenuation play a major role. Neglecting any thermal noise at the receiver is justified since interference is the main limiting factor in dense ad hoc networks.

Many analytical results are available for the case where a large number of users tries to communicate with a single central node. Thus, all users compete for the exclusive use of the channel to a specific destination. This model is well suited for the description of cellular systems where a base station has to be accessed before channel resources can be allocated to a specific user. For the simple, non–capture case, the average throughput S as a function of the offered traffic load G follows a simple exponential law $S = Ge^{-G}$ with a maximum of e^{-1} at $G = 1$. The throughput is the expected number of successful transmissions per time slot and the traffic load is the product of the transmission probability and the number of users. For the capture case and a more complex channel model, it could be shown that fading has a positive effect on the average throughput assuming that a packet will capture the channel if its received power is substantially greater than the power of the sum of all other interfering packets [14].

In ad hoc or mesh networks, things are different because there is no base station which globally manages the network traffic. Instead, there exist many pairs of communicating users trying to transmit data in a self–organized way. One of the main differences while assessing the performance of the slotted ALOHA protocol in ad hoc networks is that it is not possible to easily define a traffic load at a particular destination. There is no confined area in which users generate traffic and different users can choose different destinations. Although all users share the same channel, i.e. they use the identical resources frequency and time, several successful transmissions can occur during a single slot. This is only possible because users share resources in the spatial domain which is referred to as *spatial reuse* [15].

There is a huge body of publications dealing with the throughput analysis of the slotted ALOHA protocol in random ad hoc networks. Kleinrock *et al.*'s seminal work [16] and a later paper [17] provide first approximative solutions for the non–capture case. In the latter paper, the authors derive the following formula for the average local throughput assuming that the nodes are randomly distributed and each node selects in each slot a neighbor at random as destination (Equation (3) in [17]):

$$S = p(1-p)e^{-pN}(1-e^{-N}). \tag{1}$$

Here, p is the transmission probability and N is the number of expected neighbors of each node.

More recently, the effect of packet capture was considered in [12] which has led to insights into the performance regions of more realistic networks. However, this work addresses only transmissions over a fixed link length or transmissions to the nearest neighbor. We overcome these limitations in our model since we allow each node to randomly select one of its neighbors, i.e. the nodes which are in connection range, as desired destination. Their work, however, was valuable for the validation of our simulations.

III. Point process models

In order to study the effects of the spatial distribution of nodes in wireless ad hoc networks, we have chosen three different point process models, namely the Matérn hard–core process, the modified Thomas process and the Matérn cluster process. These processes exhibit significant differences to the purely random case, i.e. the Poisson point process (PPP), but are also suitable to model a wireless network.

The homogeneous PPP is one of the most fundamental point process models as it reflects 'complete spatial randomness' with no regularity or density trends. Its characteristic is statistical independence. Furthermore, the PPP is often used as a basis for comparison with other point processes and it is also used as a general 'building–block' for other point process models. A comprehensive overview of Poisson processes is given by Kingman in [18].

The Matérn hard–core process (MHP) is a point process where no points are allowed to lie closer together than a specific minimum distance h. Hard–core processes are widely used in theoretical physics to model dense random packing of hard spheres or for example in ecology where plants only grow in certain distances from each other. Also in the wireless communication domain it makes sense to define a certain minimum distance between any two communicating devices. For more information, the reader is referred to [19].

The modified Thomas process (MTP) [20] is one example of a Neyman–Scott process [19] which results from homogeneous independent clustering of a stationary PPP of density λ_p. The MTP process was used also in our earlier paper, where we analyzed the (partial) connectivity of ad hoc networks. The clusters themselves are independent and identically distributed point processes around the origin with a finite number of points. The construction of the MTP is fairly easy. First, the parent points are modeled as a PPP with density λ_p. Then each parent point is replaced by a cluster of points N_i. The number of points in each cluster n_i is a Poisson distributed random variable with mean μ,

$$Pr_{n_i}(n_i) = \frac{\mu^{n_i}}{n_i!} e^{-\mu}. \quad (2)$$

Inside each cluster, the cluster points with coordinates (x, y) are distributed according to a symmetric normal distribution with variance σ^2 centered at the respective parent point:

$$f_{x,y}(x, y) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right). \quad (3)$$

Another example of a Neymann–Scott process is the Matérn cluster process (MCP). As for the MTP, the parent points are distributed according to a PPP with intensity λ_p. The number of daughter points in each cluster is also Poisson with mean μ. The only difference is in the distribution of the cluster points. Where for the MTP a symmetric normal distribution is used, the cluster points of a MCP are uniformly distributed over a disc of radius R with the respective parent point as center. The parent points do not occur in the resulting point process.

We have selected the point processes described above for the existence of analytical expressions and their suitability to be used in computer simulations. The MTP can be seen as a prototypical toy–model for cellular types of hybrid ad hoc networks or sensor networks that are distributed airborne. The MHP has a clear value also as a practical model. Hard–core processes are observed in many real–world networks, since different hard–core distances are naturally imposed due to economical or human behavior reasons. All models can be conveniently described by one, two, or three parameters. These parameters can easily be tuned to achieve different models of the same class of point processes. There are thus clear individual justifications for each of the point processes we have used in our work.

To make the different models comparable, we assume in the following that all point processes have the same overall area density λ_a. The number of simulated nodes is therefore solely determined by the size of our observation window, i.e. the larger the window the more nodes we see. Table I summarizes the parameters of the used point processes. For the Poisson and Matérn hard–core process, λ is the overall node density. For both cluster processes, the MTP and the MCP, λ equals the cluster density λ_p and μ is the average number of nodes per cluster. These values are chosen to achieve a constant overall area density $\lambda_\text{a} = \lambda_\text{p}\mu = 1$. We further provide one graphical example of each of the point processes in Figure 1. In order to emphasize the difference between the two cluster processes, we have drawn dashed circles around the center of each cluster. For the MCP, the radius of the circle corresponds to the parameter R and all nodes must lie within this distance from the cluster center. For the MTP, a strict cluster border does no exist and a node is located closer to the center than R with only 95 % probability.

IV. Simulation results

In this section, we study the impact of different node distributions on the local average throughput. We compare different point process models with different sets of parameters. This gives us the chance to study both the general differences between the point process models and the impact of different parameter choices for a specific model.

TABLE I
List of point process models and parameters.

Point process model	λ	μ	σ	R	h
PPP	1	–	–	–	–
MHP 1	1	–	–	–	0.05
MHP 2	1	–	–	–	0.15
MHP 3	1	–	–	–	0.3
MTP 1	0.2608	3.8345	0.2	–	–
MTP 2	0.08516	11.743	0.35	–	–
MTP 3	0.0417	23.965	0.5	–	–
MCP 1	0.2608	3.8345	–	0.4895	–
MCP 2	0.08516	11.743	–	0.8567	–
MCP 3	0.0417	23.965	–	1.224	–

It is apparent that short range transmissions achieve the best local throughput. These transmissions occur more frequently in clustered networks. If the number of nodes in a cluster is small, we expect that the local throughput is higher than for the PPP. This is because small link lengths outweigh the effect of increased interference through peer cluster nodes. If the clusters are large, there are too many interfering nodes clumped together and the throughput decreases. With regards to [10], we conjecture that long range transmissions have a lower probability of success in clustered than in completely random networks. This is because the receiver is a part of the point process and will always belong to a cluster. Thus, it will be surrounded by close interfering nodes.

In Figure 2, we show Monte Carlo simulation results for 10 different point process models. The point processes were created using our graph generation tool GENESIS [21] and were then fed into a custom–built C simulator. Averages were taken over 100000 different representations of each point process.

We consider a Rayleigh fading channel with a deterministic distance–dependent path–loss component and a random distance–independent component. The former describes the far–field attenuation of the signal power over distance with path–loss exponent α. The latter is the channel gain and represents random signal amplitude fluctuations due to fading. Common for all plots is the path–loss exponent $\alpha = 4$, the SIR threshold $\Theta = 10\,\text{dB}$, and the network size, which is Poisson distributed with mean $N = 100$. Since the overall node density is fixed at unity, our observation window has a size of 10×10 units of area. The only varying parameter is the connection range R_c. Since absolute values do not matter, we denote by K the expected number of neighbors of a typical point of a PPP. Thus, the connection range can be calculated as $R_\text{c} = \sqrt{K/(\lambda\pi)}$. In each time slot, each node transmits with probability p to a randomly chosen neighbor. If a node is completely isolated, i.e. it has no neighbor, it remains silent. PPP results have been verified against the available analytical solutions. For non–PPP cases, there are no closed–form expressions, but other verification methods have been used, e.g. we have tested that spatial statistics measures are

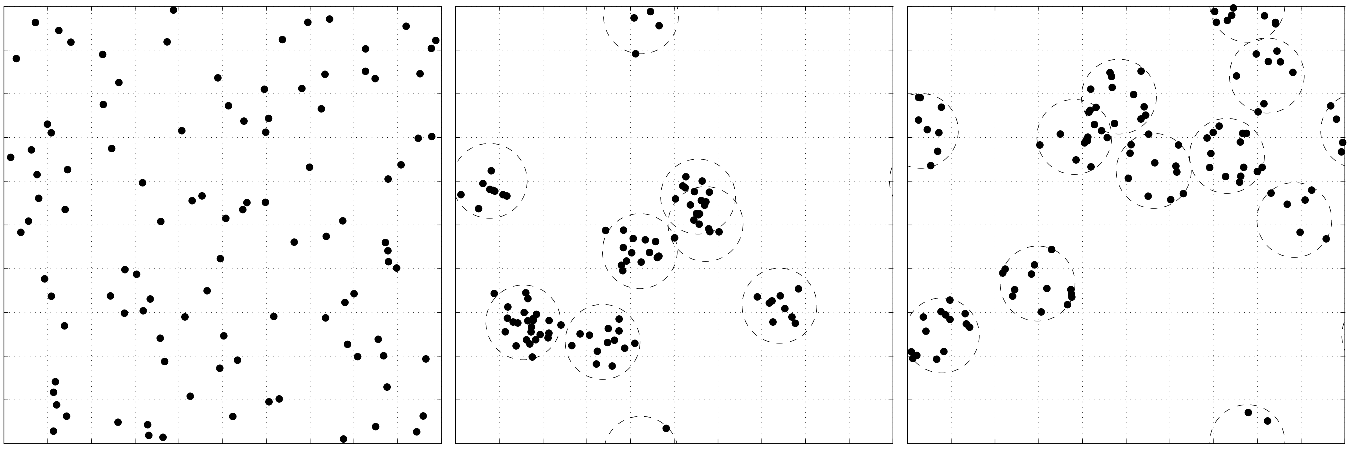

Fig. 1. Samples of different point process models.

correct for the generated point processes.

We can make the following observations: The MHPs always achieve a worse performance than the PPP. This is due the hard–core distance which does not allow for arbitrarily short links. The regularity introduced by hard–core processes is beneficial for the network connectivity. However, this regularity seems to be obstructive for the local throughput. The same observation was also made by the authors of [22] who compare the throughput of networks with random and regular topologies.

The cluster processes show clearly a different behavior than the MHPs. MTP 1 and MCP 1 achieve a higher throughput than the PPP for all connection radii. Due to the clustered distribution, the nodes are located closer to each other and, therefore, links are shorter. Due to the non–uniform distribution of nodes inside a cluster, the MTP 1 performs slightly better than the MCP 1. However, for all other parameter choices the performance of the cluster processes is inferior to the one of the PPP. We have seen that it is advantageous to have a small number of near neighbors, but that it is of great disadvantage to have too many of them. This is because larger clusters simply cause too much interference. It is interesting to observe that in this case the performance of the MCPs is better than the one of the MTPs. We refrain here from analyzing the specifics of the different cluster models more closely due to reasons of space and scope. The main aim of this paper as a subsequent companion paper to [7] is to show that the spatial distributions of nodes have strongly noticeable effects for analytical and simulation based work, and thus, also other models than the basic PPP should be considered in future studies.

V. Conclusions

We have demonstrated that non–uniformly randomly distributed node locations have a clear effect on the local throughput of ad hoc and mesh networks. The main impact of this work is that one should be careful not to make too far reaching conclusions based on simulations and analytical calculations, which are done by using simple Poisson point processes, although, in many cases, they might provide useful insights as approximations. The achievable capacity bounds should be also reported in the context of the underlying point distribution. Clustered distributions such as used here can serve as an interesting class of models in which to study different topology–dependent optimization problems.

We also argue that it is possible to leverage this information with practical communications systems. For example, recently, several groups have advocated the use of location information in the context of cognitive radios [23]. The results indicate that if the (cognitive) radio can be provided with information of its topological neighborhood, it can modify its optimization goals accordingly. For example, as the achievable capacity and connectivity are topology depended this should be taken in account when choosing the parameter values and goals for cross–layer optimization which is done by smart communication systems.

In our opinion, further work is clearly called for. We are currently working on the extension of both our analytical and Monte Carlo simulation based capacity estimates in different topologies. It is particularly interesting to extend the analysis from the local throughput analysis to full network capacity estimation and to increase the number of point distributions that are analyzed.

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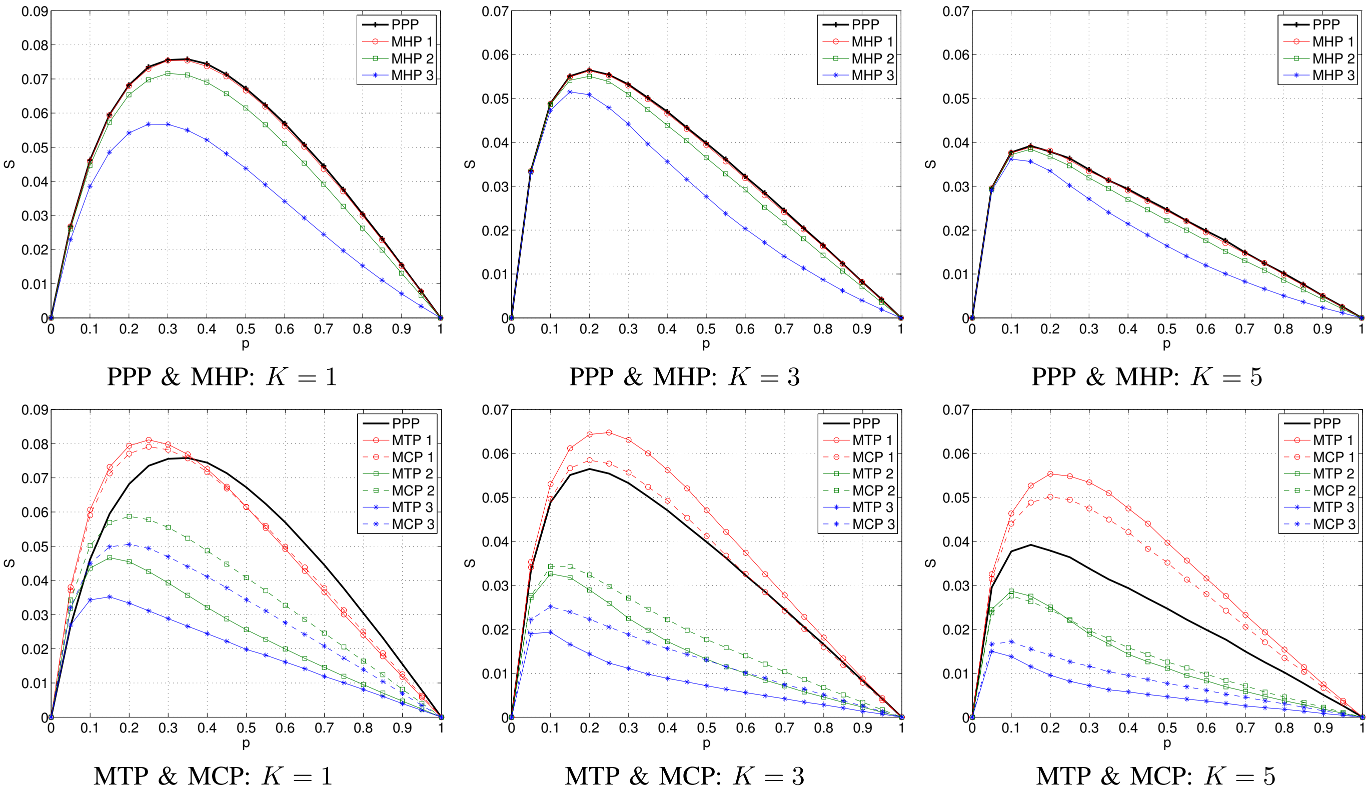

Fig. 2. Local Throughput S over transmission probability p. Average number of neighbors of the PPP $K = 1, 3,$ and 5. $N = 100$, $\alpha = 4$, and $\Theta = 10\,\text{dB}$. Transmissions over Rayleigh fading channels.

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