Proportional Fairness-Based Power Allocation and User Set Selection for Downlink NOMA Systems

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***Abstract*—The non-orthogonal multiple access (NOMA) has been investigated recently as a candidate radio access technology for the next generation mobile networks. It realizes power-domain user multiplexing by employing the successive interference cancellation (SIC). This paper focuses on the power allocation and user set selection problems in multi-user downlink NOMA systems with the aim of providing proportional fairness. The theoretical optimal power allocation solution for a candidate user set is derived in closed form. We propose a tree searching-based user set selection scheme in which the redundant user sets are removed by a delicately designed pruning method. The transmission performance of the proposed power allocation and user set selection schemes is evaluated by system-level simulations and verified to be close to the optimal value. The searching complexity for user set selection is significantly reduced compared to the exhaustive searching method. Finally, the performance gains brought by the multi-user NOMA are analyzed considering the practical application.**

***Index Terms*—Non-orthogonal multiple access, power allocation, user set selection, proportional fairness.**

I. Introduction

In order to respond to the great challenge of significantly improving system capacity in the upcoming decade, new radio access technologies are desired for the 5th generation mobile networks (5G) [1]. As a candidate technology for multi-user access, non-orthogonal multiple access (NOMA) has been investigated recently as a promising solution [2]. A novelty is to harness successive interference cancellation (SIC) to allow users being multiplexed in the power domain [3]. Hence, NOMA systems can control user multiplexing and user data rates by means of multi-user power allocation. Due to the variance of user radio links, it can achieve power domain diversity gain with appropriate power allocation (PA) and user set selection (USS) [3]–[13]. Therefore, both of the issues become significant for performance enhancement in NOMA systems.

In the literature, several power allocation schemes have been proposed for NOMA recently, which can be classified into two groups: the fixed PA and the dynamic PA. The fixed PA schemes considers the long-term-averaged channel qualities to determine the SIC order and the allocated power [6]. The power ratios allocated to the scheduled users are fixed according to the predefined parameters. On the contrary, the dynamic PA utilizes the instantaneous user channel statuses. The fractional transmit power allocation (FTPA) is a simple and well investigated dynamic PA scheme without any specific optimization objectives [2]–[5]. A general optimization objective for multiple access systems can be either improving the overall throughput or ensuring the fairness among users [9]–[11].

In order to achieve a good tradeoff between efficiency and fairness, proportional fairness (PF) has been widely accepted in multi-user access systems. In order to improve the transmission performance with the PF objective, a PA algorithm is designed in [7] based on the iterative water-filling (IWF) method. In [8], a tree-searching based transmission PA (TTPA) scheme has been proposed to reduce the complexity for PA. However, both IWF and TTPA suffer from such high computational complexity for the dynamic PA that they can hardly be applied in practice. The closed-form solution for the PF-based PA has been derived in [12] and [13]. The aim of the proposed PA method in [13] is to reduce the complexity for USS. Nevertheless, its transmission performance cannot be guaranteed without pre-optimization of the system parameters.

In [12] the authors solved the PF-based user pairing and power allocation problems in the 2-user NOMA system. In this paper, we generalize the work into the multi-user NOMA systems. First, we derive the theoretical optimal solution for the PF-based power allocation. Considering the requirement of the modulation and coding schemes (MCSs) in real systems, the candidate user sets which have the identical or similar optimal PA are omitted for USS. Based on this, we design a tree searching-based USS scheme and propose a pruning method to reduce the searching complexity. The performance of the designed PA and USS schemes is evaluated with system-level simulations and compared to the optimal one which is obtained by exhaustive searching. The computational complexity and practicability are also analyzed.

The rest of the paper is organized as follows. Section II describes the system model of the SIC-based NOMA system. In section III, the closed-form solution for PF-based PA is derived along with the conditions for its uniqueness. A tree searching-based USS scheme is designed in section IV. In section V, the simulation results of the proposed scheme are presented and analyzed. Finally, conclusions are drawn in section VI.

II. System Model

We consider one cell in a downlink cellular network. Its serving user index set is denoted as $\mathbf{U}_M = \{1, 2, ..., M\}$. The frequency bandwidth is divided into K resource blocks (RBs) with indices $k \in [1, K]$ as in the orthogonal frequency-division multiple access (OFDMA) systems [14]. Successive interference cancellation is applied in the NOMA system to allow superposition of signals with different transmit power levels for multiple users [2]. We denote the set of users served simultaneously in RB k as $\mathbf{U}_k = \{l_k(1), ..., l_k(n), ..., l_k(N_k)\}$, where $l_k(n)$ is the user index of the n-th scheduled user in the RB. The number of users in $\mathbf{U}_k$, i.e., $N_k = |\mathbf{U}_k|$, is assumed no larger than a limitation $N_{\max}$ due to the processing complexity for SIC receivers.

Both transmission and reception use single-antenna systems. Hence, the received signal in RB k at its n-th served user is given as

$$y_{l_k(n)} = h_{l_k(n)} \sum_{i=1}^{N_k} s_{l_k(i)} \sqrt{P_{l_k(i)}} + I_{l_k(n)} + \sigma_{l_k(n)}, \tag{1}$$

where $h_{l_k(n)}$ is the comprehensive channel gain of path loss, shadow fading, and fast fading in amplitude. $s_{l_k(i)}$ denotes the data symbol with a unit mean power, i.e., $\mathbb{E}[\left|s_{l_k(i)}\right|^2] = 1$. $P_{l_k(i)}$ denotes the transmit power allocated to user $l_k(i)$. Symbol $I_{l_k(n)}$ denotes the overall inter-cell interference signals, and $\sigma_{l_k(n)}$ is the additive white Gaussian noise.

All the RBs available in a cell are assumed to share the transmit power equally. Thus, $P_{l_k(n)}$ is expressed as

$$P_{l_k(n)} = \lambda_{l_k(n)} P_t / K, \quad n = 1, ..., N_k, \tag{2}$$

where P_t is the total transmit power, and $\lambda_{l_k(n)}$ is the power ratio assigned to user $l_k(n)$, which satisfies

$$\sum_{n=1}^{N_k} \lambda_{l_k(n)} = 1, \quad \lambda_{l_k(n)} \in (0, 1). \tag{3}$$

We denote the channel quality indicator (CQI) of user $l_k(n)$ as $\phi_{l_k(n)}$. It is defined as the signal-to-interference-plus-noise ratio (SINR) with the full transmit power of a RB allocated to the user link. Hence, $\phi_{l_k(n)}$ is calculated as

$$\phi_{l_k(n)} = \frac{P_t \left|h_{l_k(n)}\right|^2}{K P_{IN, l_k(n)}}, \tag{4}$$

where $P_{IN, l_k(n)}$ denotes the sum power of inter-cell interference and noise received by user $l_k(n)$.

In downlink NOMA systems, SIC is carried out in user receivers for decoding. We assume that the scheduled users in a RB are sorted in the descending order in terms of their CQIs, and user $l_k(n)$ can decode and cancel successively the interference signals of user $l_k(n+1) \sim l_k(N_k)$ by SIC [4]. The post-processing SINR (PSINR) of user $l_k(n)$ after SIC is

$$\Phi_{l_k(n)} = \frac{\phi_{l_k(n)} \lambda_{l_k(n)}}{\phi_{l_k(n)} \sum\limits_{i=1}^{n-1} \lambda_{l_k(i)} + 1}. \tag{5}$$

For implementation of SIC, a user with better channel quality, namely, higher $\phi_{l_k(n)}$, has to be informed the MCSs and the power ratios allocated to the users who have lower CQIs. Then, user $l_k(n)$ needs to decode the signals and to cancel the interference of user $l_k(n+1) \sim l_k(N_k)$ in the reverse order [4]. If N_k is large, the signaling cost and the processing complexity at user equipment become very high.

III. PF-Based Power Allocation for NOMA

In this section, we firstly introduce the proportional fairness scheduling policy and present the PF scheduling factor. Based on this, we derive the theoretical optimal PF-based PA for a candidate user set and demonstrate the conditions for its uniqueness. The MCS limits in real systems are taken into account to reduce the number of compared user sets for USS.

A. Proportional Fairness Policy

PF is able to maximize the geometric mean of user rates such that both efficiency and user fairness can be guaranteed [15]. Therefore, it has been adopted as the objective of PA and USS for NOMA systems in several works [7], [8], [12], [13]. PF considers the instantaneous user data rate along with the long-term-averaged rate, which is calculated as

$$R_m(t+1) = \left(1 - \frac{1}{t_c}\right) R_m(t) + \frac{1}{t_c} \sum_{k=1}^{K} x_{m,k}(t)\, r_{m,k}(t),$$
$$k = 1, ..., K, \quad m = 1, ..., M, \tag{6}$$

where t_c is the averaging window size, and $x_{m,k}(t)$ is the scheduling 0-1 index. If user m in RB k at t-th frame is scheduled, it equals to 1, otherwise, 0. $r_{m,k}(t)$ is the obtainable data rate of user m in RB k at t-th frame and is calculated by

$$r_{m,k}(t) = N_{sc} S_e \eta_{m,k}(t) / T_s = r_0 \eta_{m,k}(t), \tag{7}$$

where N_{sc} is the number of subcarriers in each RB, S_e is the number of effective symbols in one frame, and T_s is the frame duration [14]. The achievable spectrum efficiency, $\eta_{m,k}$, is calculated according to the selected MCS and the expected block error rate (BLER) [16].

According to [7] and [12], the scheduling factor for the PF-based NOMA system is given as follows.

$$\omega_k(t) = \sum_{m=1}^{M} x_{m,k}(t) \left[\frac{r_{m,k}(t)}{R_m(t)}\right], \quad k = 1, ..., K. \tag{8}$$

In order to maximize the geometric mean of the long-term-averaged user rates, ω_k needs to be maximized with appropriate selected user set and power allocation.

B. Optimal PF-Based Power Allocation

We sort the users in a candidate user set for SIC in the descending order of their CQIs. The sequence is denoted as

$$\mathbf{S}_C = \{c(1), \ldots, c(S)\}, \quad S = |\mathbf{S}_C|,$$

where $\phi_{c(i)} > \phi_{c(i+1)}, i = 1, \ldots, S-1$. The cumulative power ratio (CPR) allocated to the first i users is denoted as

$$\alpha_{c(i)} = \sum_{j=1}^{i} \lambda_{c(j)}, \quad i = 1, \ldots, S.$$

According to (3), $\alpha_{c(S)} = 1$. Without losing generality, we define $\alpha_{c(0)} = 0$ for ease of expression in the following.

Focusing on the optimization of PA for user sequence $\mathbf{S}_C$, we neglect the marks t and k in (8). Hence, it can be any one RB to be optimized. Let $x_{m,k} = 1$ for all $m \in \mathbf{S}_C$, and other $x_{m,k} = 0$. Then, the PF scheduling factor can be rewritten as a function of CPRs, i.e.,

$$\omega\left(\alpha_{c(1)}, \ldots, \alpha_{c(S-1)}\right) = \sum_{i=1}^{S} \frac{r_{c(i)}\left(\alpha_{c(i-1)}, \alpha_{c(i)}\right)}{R_{c(i)}}, \quad (9)$$

where $r_{c(i)}\left(\alpha_{c(i-1)}, \alpha_{c(i)}\right)$ is the instantaneous obtainable data rate of user $c(i)$. Relaxing the MCS constraint in (7) and using the Shannon capacity to calculate the achievable symbol bitrate, we have

$$r_{c(i)}\left(\alpha_{c(i-1)}, \alpha_{c(i)}\right) = r_0 \log_2\left(\frac{1+\alpha_{c(i)}\phi_{c(i)}}{1+\alpha_{c(i-1)}\phi_{c(i)}}\right). \quad (10)$$

Combining (9) and (10), we have the PF scheduling factor as follows.

$$\omega\left(\alpha_{c(1)}, \ldots, \alpha_{c(S-1)}\right) = \frac{r_0 \log_2\left(1+\phi_{c(S)}\right)}{R_{c(S)}} + \quad (11)$$
$$r_0 \sum_{i=1}^{S-1}\left[\frac{\log_2\left(1+\alpha_{c(i)}\phi_{c(i)}\right)}{R_{c(i)}} - \frac{\log_2\left(1+\alpha_{c(i)}\phi_{c(i+1)}\right)}{R_{c(i+1)}}\right]$$

The factor is related to $(S-1)$ CPRs which can be optimized independently according to (11).

The partial derivative of the scheduling factor is derived as

$$\begin{aligned} {\omega'}_{c(i)}\left(\alpha_{c(i)}\right) &\triangleq \frac{\partial \omega}{\partial \alpha_{c(i)}} \quad (12) \\ &= \frac{r_0}{\ln 2}\left[\frac{\phi_{c(i)} R_{c(i)}^{-1}}{\left(1+\alpha_{c(i)}\phi_{c(i)}\right)} - \frac{\phi_{c(i+1)} R_{c(i+1)}^{-1}}{\left(1+\alpha_{c(i)}\phi_{c(i+1)}\right)}\right]. \end{aligned}$$

With different parameters, the scheduling factor function results in three cases as follows.

Case 1: When the parameters of two adjacent user $c(i)$ and $c(i+1)$ satisfy the following condition, the function has one and only one stationary point in the region $(0, 1)$.

$$\frac{\phi_{c(i+1)}}{\phi_{c(i)}} < \frac{R_{c(i+1)}}{R_{c(i)}} < \frac{1+\phi_{c(i)}^{-1}}{1+\phi_{c(i+1)}^{-1}} \quad (13)$$

The stationary point is

$$\alpha_{c(i)}^{*} = \frac{R_{c(i+1)}\phi_{c(i+1)}^{-1} - R_{c(i)}\phi_{c(i)}^{-1}}{R_{c(i)} - R_{c(i+1)}}. \quad (14)$$

For the integrity of the symbol definition, we denote

$$\alpha_{c(0)}^{*} = 0, \quad \alpha_{c(S)}^{*} = 1. \quad (15)$$

The second derivative of the factor function at the stationary point is calculated as follows.

$$\begin{aligned} {\omega''}_{c(i)}\left(\alpha_{c(i)}^{*}\right) &\triangleq \left.\frac{\partial^2 \omega}{\partial \alpha_{c(i)}^2}\right|_{\alpha_{c(i)}=\alpha_{c(i)}^{*}} \\ &= \frac{r_0 \phi_{c(i)}^2 \phi_{c(i+1)}^2 \left(R_{c(i+1)} - R_{c(i)}\right)^3}{\ln 2 R_{c(i)}^2 R_{c(i+1)}^2 \left(\phi_{c(i)} - \phi_{c(i+1)}\right)^2} < 0 \end{aligned}$$

So $\alpha_{c(i)}^{*}$ is the only maximum point in the region $(0, 1)$. Due to that the factor is a continuous derivable function with respect to $\alpha_{c(i)}$, it holds that

$${\omega'}_{c(i)}\left(\alpha_{c(i)}\right) > 0, \quad \alpha_{c(i)} \in \left(0, \alpha_{c(i)}^{*}\right), \quad (16)$$

$${\omega'}_{c(i)}\left(\alpha_{c(i)}\right) < 0, \quad \alpha_{c(i)} \in \left(\alpha_{c(i)}^{*}, 1\right). \quad (17)$$

We denote the optimal CPRs as $\hat{\alpha}_{c(i)}, i = 1, ..., S-1$, which result in the maximum PF scheduling factor. In any case, $\hat{\alpha}_{c(0)} = 0$ and $\hat{\alpha}_{c(S)} = 1$. The optimal power ratios are

$$\hat{\lambda}_{c(i)} = \hat{\alpha}_{c(i)} - \hat{\alpha}_{c(i-1)}, \quad i = 1, ..., S. \quad (18)$$

If there exists $\hat{\lambda}_{c(i)} = 0$, the optimal PA for $\mathbf{S}_C$ is identical with one of its proper subsequences $\mathbf{S}_C / \{c(i)\}$. The uniqueness of the optimal PA for a candidate user set is defined as follows.

Definition 1. *If a user sequence has its optimal PA different from all of its proper subsequences, equally, every user in it has positive allocated power , it is defined as a singular user sequence. Otherwise, it is nonsingular.*

Theorem 1. *The equivalent condition for a user sequence $\mathbf{S}_C$ to be singular is*

$$\alpha_{c(i)}^{*} > \alpha_{c(i-1)}^{*}, \quad i = 1, ..., S. \quad (19)$$

Proof. Sufficiency: Let a possible solution of PA be $\alpha_{c(i)}^{*}, i = 1, ..., S-1$. If (19) holds, each user has allocated power as

$$\lambda_{c(i)}^{*} = \alpha_{c(i)}^{*} - \alpha_{c(i-1)}^{*} > 0, \quad i = 1, ..., S.$$

Since $\alpha_{c(i)}^{*}$ is the maximum point of the factor function, it must be the optimal solution. Hence, the user sequence is singular.

Necessity: Assuming that user $c(i-1)$ and $c(i)$ have $\alpha_{c(i)}^{*} \leq \alpha_{c(i-1)}^{*}$, the optimal solution must have $\hat{\alpha}_{c(i-1)} = \hat{\alpha}_{c(i)}$ due to the characteristics of the factor function as in (16) and (17). Thus, $\hat{\lambda}_{c(i)} = 0$. The user sequence is nonsingular with the assumption $\alpha_{c(i)}^{*} \leq \alpha_{c(i-1)}^{*}$. Therefore, a singular user sequence must satisfy (19). □

Corollary 1. *A user sequence $\mathbf{S}_C$ has more than 1 user, i.e., $S = |\mathbf{S}_C| > 1$. If $\mathbf{V}_C = \mathbf{S}_C / \{c(S)\}$ is nonsingular, then $\mathbf{S}_C$ is nonsingular.*

Proof. If $\mathbf{V}_C = \mathbf{S}_C / \{c(S)\}$ is nonsingular, by Theorem 1 there must exist at least one $i \in \{1, ..., S\}$ holding $\alpha_{c(i)}^{*} \leq \alpha_{c(i-1)}^{*}$. This case also exists in $\mathbf{S}_C$, thus it is nonsingular. □

Case 2: When there exist two adjacent users satisfying

$$\frac{R_{c(i+1)}}{R_{c(i)}} \leq \frac{\phi_{c(i+1)}}{\phi_{c(i)}}, \quad (20)$$

it holds

$${\omega'}_{c(i)}\left(\alpha_{c(i)}\right) < 0, \quad \alpha_{c(i)} \in (0, 1).$$

Thus the scheduling factor is a monotonic decreasing function with respect to $\alpha_{c(i)}$ and there is no stationary point in the region $\alpha_{c(i)} \in (0, 1)$. So condition (19) is not satisfied and $\mathbf{S}_C$ is nonsingular by Theorem 1.

Case 3: When two adjacent users satisfy

$$\frac{R_{c(i+1)}}{R_{c(i)}} \geq \frac{1+\phi_{c(i)}^{-1}}{1+\phi_{c(i+1)}^{-1}}, \tag{21}$$

it holds

$${\omega'}_{c(i)}\left(\alpha_{c(i)}\right) > 0, \quad \alpha_{c(i)} \in (0,1).$$

As in Case 2, there is no stationary point in the region $\alpha_{c(i)} \in (0,1)$. So $\mathbf{S}_C$ is nonsingular.

From the three cases above, we can conclude that (13) in Case 1 is a necessary condition for a user sequence to be singular. So far, we have the theoretical optimal PA solution of a singular user sequence without considering the MCS constraint, i.e.,

$$\hat{\alpha}_{c(i)} = \alpha^*_{c(i)}, \quad i = 1, ..., S-1. \tag{22}$$

A nonsingular user sequence is not necessary to be considered for the user set selection because at least one of its proper subsequences must have the identical optimal solution of PA.

C. Power Allocation with MCS Limits

Considering the MCS selection in real systems, the PSINR has to meet the minimum requirement, which is denoted as β. If the PSINR of a user is lower than β, the transmission fails due to high BLER. Therefore, PSINRs need to satisfy

$$\Phi_{c(i)} \geq \beta, \quad i = 1, ..., S. \tag{23}$$

Theorem 2. *With a given parameter $\beta > 0$, if $\mathbf{S}_C$ is singular, its user CQIs must satisfy*

$$\phi_{c(i)} \geq \beta, \quad i = 1, ..., S. \tag{24}$$

Proof. Denoting the optimal solution of CPRs while $\beta > 0$ as $\tilde{\alpha}_{c(i)}, i = 1, ..., S$, and substituting them and (5) into (23), we have

$$\phi_{c(i)} \geq \beta\left[\tilde{\alpha}_{c(i)} - \tilde{\alpha}_{c(i-1)}(1+\beta)\right]^{-1}, \\ \tilde{\alpha}_{c(i)} > \tilde{\alpha}_{c(i-1)}(1+\beta).$$

Since $\tilde{\alpha}_{c(i-1)}, \tilde{\alpha}_{c(i)} \in (0,1)$, (24) holds. □

We further divide the singular user sequences into two kinds which are defined as follows.

Definition 2. *With a given parameter $\beta > 0$, if the optimal PA of a singular user sequence $\mathbf{S}_C$ satisfies*

$$\tilde{\alpha}_{c(i)} = \alpha^*_{c(i)}, \quad i = 1, ..., S-1, \tag{25}$$

$\mathbf{S}_C$ is a singular user sequence of the first kind (S1). Otherwise, it is a singular user sequence of the second kind (S2).

Combining (5) and (23) yields

$$\alpha_{c(i)} \geq (1+\beta)\,\alpha_{c(i-1)} + \beta\phi_{c(i)}^{-1} > \alpha_{c(i-1)}. \tag{26}$$

Accordingly, we denote $\alpha^l_{c(i)}$ as follows,

$$\alpha^l_{c(i)} \triangleq (1+\beta)\,\alpha^*_{c(i-1)} + \beta\phi_{c(i)}^{-1} > \alpha^*_{c(i-1)}. \tag{27}$$

Theorem 3. *The equivalent condition for a candidate user sequence $\mathbf{S}_C$ to be S1 is*

$$\alpha^l_{c(i)} \leq \alpha^*_{c(i)}, \quad i = 1, ..., S. \tag{28}$$

Proof. Sufficiency: Let a possible solution of PA be $\alpha^*_{c(i)}, i = 1, ..., S-1$. If (28) holds, according to (27), we have

$$\alpha^*_{c(i-1)} < \alpha^l_{c(i)} \leq \alpha^*_{c(i)} < \alpha^l_{c(i+1)} \leq \alpha^*_{c(i+1)}, \\ i = 1, ..., S-1.$$

By Theorem 1, (19) holds and $\mathbf{S}_C$ is singular. Since $\alpha^*_{c(i)}$ is the maximum point, (25) holds. Hence, $\mathbf{S}_C$ is S1.

Necessity: Assuming that user $c(i-1)$ and $c(i)$ in a S1 user sequence have $\alpha^l_{c(i)} > \alpha^*_{c(i)}$, the optimal solution is $\tilde{\alpha}_{c(i)} = \alpha^l_{c(i)} > \alpha^*_{c(i)}$ due to that the factor function is monotonic decreasing as in (17). This conflicts with (25). Hence, the assumption does not hold and a S1 user sequence must satisfy (28). □

Corollary 2. A user sequence $\mathbf{S}_C$ *has more than 1 users, i.e., $S = |\mathbf{S}_C| > 1$. If $\mathbf{V}_C = \mathbf{S}_C / \{c(S)\}$ is not S1, then $\mathbf{S}_C$ is not S1.*

Proof. If $\mathbf{V}_C = \mathbf{S}_C / \{c(S)\}$ is not S1, there must exist $\alpha^*_{c(i)} < \alpha^l_{c(i)}$ in $\mathbf{S}_C$. This case also exists in $\mathbf{S}_C$ such that by Theorem 3 it cannot be a S1 user sequence. □

From the proof of Theorem 2, we can find that the optimal CPR $\tilde{\alpha}_{c(i)}$ in a S2 user sequence is related to $\alpha^l_{c(i-1)}$. As in (27), $\alpha^l_{c(i)}$ is dependent on $\alpha^*_{c(i-1)}$. In this case, the CPR variables are not independent during the optimization of a S2 user sequence. Thus a S2 user sequence has no concise closed-form solution for the optimal PA as S1 unless it has no more than 2 users [12]. Due to this fact, the S2 user sequences are ignored in our proposed USS scheme.

Neglecting S2 user sequences may cause performance loss since their optimized scheduling factors are possibly higher. However, if β is small enough, $\alpha^l_{c(i)}$ is close to $\alpha^*_{c(i-1)}$. Thus, when $\alpha^*_{c(i)} < \alpha^l_{c(i)}$ in a S2 user sequence, there exists $\alpha^*_{c(i-1)} \approx \alpha^*_{c(i)} \approx \alpha^l_{c(i)}$ in practice. Therefore, $\lambda^*_{c(i)} \approx 0$ and the optimal PA for a S2 user sequence is similar with a nonsingular user sequence that can be omitted for USS.

IV. Tree Searching-Based User Set Selection

According to the closed-form PA solution and the conditions for its uniqueness in section III, we design a user set selection scheme based on a tree searching method in this section.

A. Construction of the User Set Trees

We firstly define a forest with multiple trees which include all possible use sets for scheduling. The forest is constructed according to the following rules:

- Each tree root node has only one unique user in $\mathbf{U}_M$.
- Each child node has a unique candidate user set in which users are sorted into a descending sequence in terms of their CQIs.
- The child nodes of a node which has user sequence $\mathbf{S}_C = \{c(1), \ldots, c(S)\}$ are
$$\left\{\mathbf{S}_C \cup \{m\}\,|\phi_m < \phi_{c(S)}, m \in \mathbf{U}_M\right\}.$$
- The level of a node with user sequence $\mathbf{S}_C$ equals to its size S and it satisfies $S \leq N_{\max}$.

A 4-user example with $N_{\max} = 3$ is presented in Fig. 1.

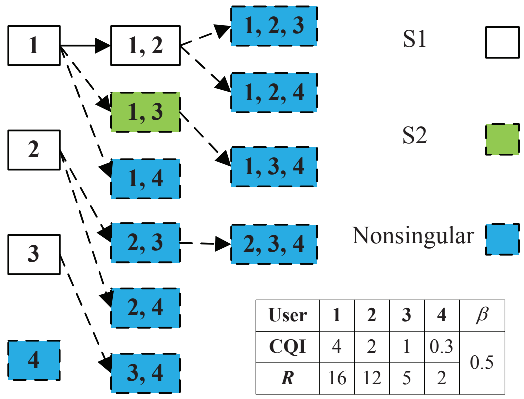

Fig. 1. Illustration of the user set tree structures. ($M = 4$, $N_{\max} = 3$)

TABLE I
SIMULATION PARAMETERS

Parameter	Value
Inter site distance	500 m
Minimum distance from a user to BS	20 m
Bandwidth	10 MHz @ 1800 MHz
BS transmit power (P_t)	46 dBm
BS transmit antenna gain	18 dBi
Path loss	138.47+38.22log(d[km])
Standard deviation of shadow fading	8 dB
Fast fading	Rayleigh model
Noise power density	-174 dBm/Hz
Noise figure	9 dB
Number of RBs (K)	50
Number of subcarriers (N_{sc})	12 per RB
Number of effective symbols (S_e)	10 per frame
Frame duration (T_s)	1 ms
Averaging window size (t_c)	100

B. Proposed User Set Selection Scheme

A straightforward method to find the optimal scheduled user set is exhaustive searching [7], [8]. All the nodes need to be traversed to compare their PF scheduling factors with the optimized PA.

However, according to our analysis in Section III, only the nodes with S1 user sequences are considered and other nodes are pruned to reduce the amount of the compared nodes. By Corollary 1 and 2, a child node cannot have S1 user sequence if its parent has a S2 or nonsingular user sequence. Thus, we traverse the forest with the depth-first principle and prune S2 and nonsingular nodes along with all their descendants.

First, in order to reduce the trial amount, Theorem 2 is used to remove the user sets that contain invalid users. Then, the user sequence is tested by Theorem 1 and 3. In this way, the computational complexity for candidate user set comparison can be significantly reduced. A 4-user pruning example is given in Fig. 1.

V. SIMULATIONS AND RESULTS

In this section the performance of our proposed PA and USS schemes is evaluated with system-level simulations. A downlink cellular network with 37 cells is deployed in a hexagonal grid pattern. The simulation parameters are listed in Table I [14]. To avoid the edge effect, only the performance of the central cell is computed and other 36 BSs act as interferers. User terminals are uniformly randomly distributed. The MCSs are chosen according to [14]. The PSINR threshold β is -9.478 dB for the target BLER lower than 0.1 [16]. The long-term-averaged user rates are initialized randomly, and we compute the performance statistics over 5,000 frames. This is done after 1,000 initial frames to ensure that the system status is stable for performance calculation. The evaluated performance is compared with the optimal one which is obtained by exhaustive searching for both PA and USS. The exhaustive searching granularity of power ratios is set to be 0.01 [8].

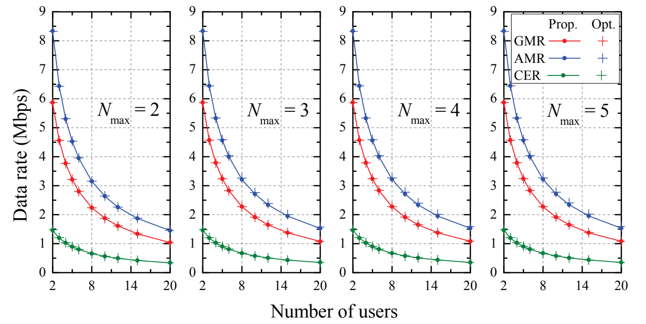

Fig. 2. Comparison of the throughput performance of the proposed scheme with the optimal value.

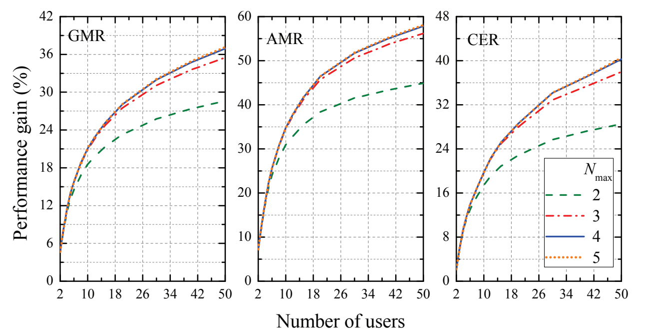

Fig. 3. Performance gains of the NOMA system over OFDMA.

Fig. 2 presents the performance of geometric mean rate (GMR), arithmetic mean rate (AMR) and cell-edge user rate (CER). CER is defined as the time-averaged user rate at the 5th percentile of the lowest ones. The simulation results reveal that the performance of our proposed scheme is identical with the optimal one while $N_{\max} = 2$. When $N_{\max}$ is larger than 2, there is only slight performance loss. As the number of users rises, the performance decreases because there are fewer RBs obtained per user.

We calculate the performance gain of the NOMA system over OFDMA as shown in Fig. 3. The performance gain increases while there are more users in the cell, owing to the power domain user diversity gain brought by the PF scheduling. As $N_{\max}$ increases, more users are allowed to be simultaneously scheduled per RB such that the diversity gain increases with more candidate user sets.

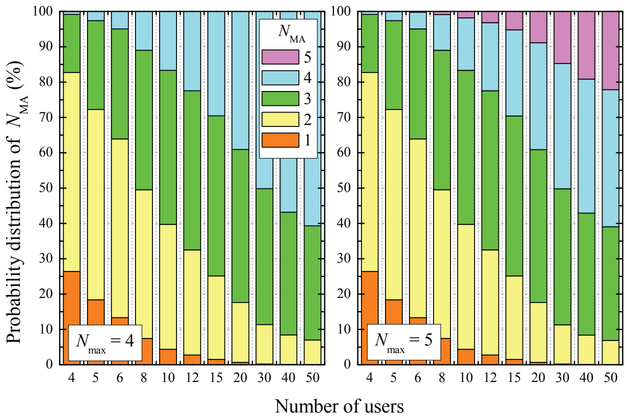

Fig. 4. Probability distribution of the multiplexed user numbers per RB.

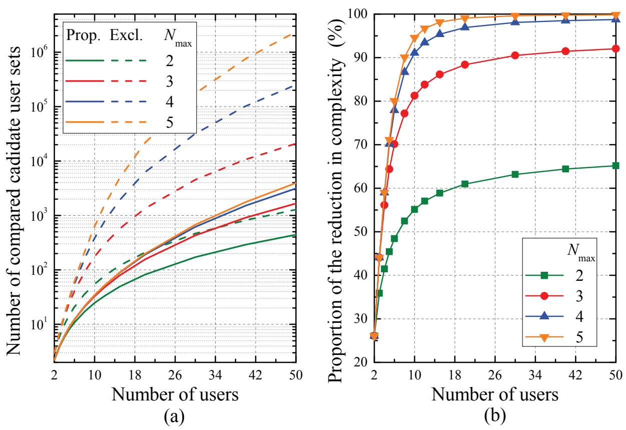

Fig. 5. The average number of compared candidate user sets per RB and its reduced proportion by the proposed USS scheme.

We denote the number of simultaneously multiplexed users in a RB as N_{MA} and compute the probability distribution of N_{MA} while $N_{\max}$ equals to 4 and 5, as shown in Fig. 4. As the number of users in the cell rises, the probabilities of $N_{\mathrm{MA}} = 4$ and $N_{\mathrm{MA}} = 5$ increase and their sum is more than half when there are more than 30 users in the cell. This is due to that more users in the cell increase the chance to select the user sets with 4 or 5 users for scheduling. However, as shown in Fig. 3, the additional performance gain brought by increasing $N_{\max}$ to 4 or 5 is very limited. Considering the signaling overhead and decoding complexity for high-order SIC, the benefit is limited in practice to use the NOMA with $N_{\max} \geq 4$.

The average number of the compared candidate user sets per RB is presented in Fig. 5 (a). The proposed USS scheme results in much smaller numbers of the compared user sets than exhaustive searching, especially while there are more users. Hence, the pruning method can efficiently reduce the computational complexity for USS. We calculate the proportion of the reduction as shown in Fig. 5 (b). The complexity reduction is higher while $N_{\max}$ is larger. For instance, when $N_{\max} = 5$, the reduced proportion reaches 99% with more than 20 users. While there are more than 8 users, at least half candidate user sets can be omitted by our proposed USS scheme.

VI. Conclusion

In this paper we have focused on the power allocation and user set selection problems for downlink NOMA. We proposed a tree searching-based USS scheme based on the closed-form solution of PA with the PF objective. Only S1 user sequences are considered as valid USS candidates while the S2 and nonsingular ones are avoided. Simulation results indicated that the proposed PA and USS schemes achieve very close performance with the optimal one in terms of both efficiency and user fairness. In addition, the computational complexity is significantly reduced compared to the exhaustive searching method. Considering the additional cost in transceivers and system optimization, the NOMA systems with 2 or 3 simultaneously multiplexed users are practical in reality.

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